THE UNIVERSITY OF ALBERTA

SIMULATION OF LAKE LEVELS
IN THE
COOKING LAKE MORAINE

by

© DAVID CRAWFORD

A THESIS

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ABSTRACT

Natural variability in mean temperatures and total precipitation has been related to fluctuations in the levels of lakes contained within the Cooking Lake Moraine area.

The monthly lake levels have been estimated by developing a water-balance model from historical meteorological data recorded at Edmonton, physical characteristics of the basin and lake level records of Cooking Lake. The technique devised by Thornthwaite, et al, has been used to obtain the best estimate of evapotranspiration and surface runoff, the major components of the model.

This thesis shows that lake level fluctuations can be related to natural variations in meteorological variables, by generating a long record of meteorological data using Monte Carlo Methods, and by obtaining simulated lake levels using the water-balance model developed.

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CHAPTER I

INTRODUCTION

The objective of this study is to develop a mathematical model which will describe the stochastic nature of the water balance in the Cooking Lake Moraine area, located approximately eighteen miles south-east of Edmonton. Figure 1 is a map of the area showing the main lakes of the system and the approximate flow path of streams connecting the lakes. The water balance model will be developed from a consideration of the available data and of the variables which effects directly or indirectly the input or output of water contained within the lake system.

An experimental statistical technique will then be used to generate a series of psuedo random numbers with a given distribution function. The given distribution function will either be the same as or within acceptable limits as that of the observed variable. The generated values will be used as the input variables of the water balance model and an investigation made of the expected fluctuations in the levels of the lakes contained within the boundaries of the Cooking Lake Moraine. The results of this study are applicable in a broad sense to all lakes in the same climatological region.

Background Information on the Area

In 1970 a petition, signed by local citizens of the Cooking and Hastings Lake area, called for Provincial Government action to reclaim the Beaverhill Watershed. As a direct result of this petition the Province organised and financed a multi-disciplined study of the entire

FIGURE 1: Location of Cooking Lake Moraine with respect to Edmonton

Cooking Lake Moraine to determine its potential as a recreational area for the City of Edmonton and as a standard for other conservation areas within Alberta. [Stanley Associates, 1974. EPEC Consulting, 1971, 1976.]

The prime location of the area, Figure 1, makes it an invaluable source of water based recreational activities. This potential has been recognised since the first settlers moved into the area and the first recreation establishment was the Goney Island Sporting Club formed in 1894. This club was basically a rather exclusive organisation available to only the more affluent members of the community. It was not until 1909, when the first passenger train of the Grand Trunk Pacific Co. ran passed the north shore of Cooking Lake, did the area become available to the working classes of Edmonton.

The area is generally known as the Cooking-Lake Moraine but it also has several alternative names, principally Beaverhill Watershed area, and its old Indian name Amisk Wuchee (Beaver Hills). In whatever case it still comprises an integral series of lakes believed to be dependent to various extents on each other.

The topography of the area is characteristic of hummocky disintegration moraine. This glacial moraine was formed during and after the last glacial period of about 9000 years ago. The level of the area is generally above that of the surrounding land making a watershed area of approximately 1500 square miles, draining from the south into the North Saskatchewan River via Beaverhill Lake.

Early descriptions of the lake system and its surrounding area mentions the abundance of pelican, cormorants and blue heron nests, which implies a large fish population, thousands of deer, elk and moose

and dense spruce forests surrounding clear lakes connected by streams suitable for large canoes (Nyland, 1969). The past eighty years has seen this description change to one where the wildlife has been decimated, the forests cleared and the lakes, in certain cases, are mere sloughs.

The Beaver Hills were invaded around 1780 by the Blackfeet during their migration from Eastern Canada. They, in turn, had to yield to the Cree. Around 1890 the influences of the first white settlers began to be felt. Almost immediately upon their arrival to the area the settlers began to clear the forests, with fires, so that cultivation of the land could proceed. Some of these fires burned out of control due to dry ground conditions, high winds or the over zealousness of the fire setters. In several cases large areas of prime forest were destroyed, the most extensive occurring in 1895. This fire destroyed an area of forest bounded by Edmonton in the west, Beaverhill Lake in the east, Fort Saskatchewan in the north and Cooking Lake in the south.

A particularly important event, with respect to the water balance of the area, was the opening of a canal from Miquelon Lake to a reservoir serving the City of Camrose in 1927. Up until the opening, of the canal, the lakes were in good condition but the opening year and the first years of operation coincided with relatively dry years and a drastic drop in the level of Miquelon Lake occurred. Three shallower lakes formed from Miquelon Lake after this drop. The lakes downstream of Miquelon Lake also appear to have been affected. It has been reported (Nyland, 1969) that in the dry years following the opening of the canal the water levels in Oliver, Ministik and Cooking Lakes were lower than anyone could remember.

Although the canal is no longer in use and has been blocked off,

Miquelon Lake has been unable to recover naturally to its former regime. It would therefore appear that 1927 marks an obvious turning point in the water balance of Miquelon Lake and possibly the entire lake system.

Activities such as land clearing by fire setting, water diversions and changes in land use have all played their respective roles in the decline of the lake system. It is the purpose, however, of this treatise to investigate the important role nature plays in the water balance of the system, in particular, the stochastic variations in precipitation, evaporation and evapotranspiration, and to determine if natural variability in weather could have caused the large fluctuations in lake levels that have been observed.

Surficial and Bedrock Geology

The topography associated with hummocky disintegration moraine characterises the Cooking Lake Moraine area. Hummocky moraine, by definition, has a local relief of more than fifteen feet and is generally thick (40 to 150 feet). [Bayrock and Hughes, 1962. Bayrock, 1972]. The composition of the moraine is primarily of till made up of almost equal proportions of sand, silt and clay containing pebbles and boulders. Some lenses of sand, gravel and local bedrock are also to be found in the surficial deposits. Recent lacustrine deposits have been mapped around Cooking, Joseph and Miquelon Lakes and it is believed that the other lakes in the system will show the same general deposits. [Bayrock, 1972]. The lacustrine deposits are predominately composed of silt and clay with local marl deposits, with some clean sand found in places.

Several geological cross-sections and surficial features are defined in Figure 2 and the cross-sections shown in Figure 3. [Stanley & Associates, 1974].

The surficial deposits described above overlie the Upper Cretaceous, Horseshoe Canyon formation of the Edmonton group. The Horseshoe Canyon formation is underlayin by the shaley beds of the Bearpaw formation which is in turn underlayin by the Belly or Judith River formation. The Belly formation has a lithology similar to that of the Edmonton group. [Carlson, 1966].

The main features of the bedrock topography are a general coincidence of preglacial and present day water divides, and major buried valleys following the general trend of Hastings and Katchemut Creek. [Carlson, 1966] There have been two branches of the Vegreville bedrock channel identified in the area. Ministik lake lies over one of these branches and Cooking Lake over the other [Farvolden, 1963. Carlson, 1966]. Sand and gravel in places, exceeding thirty feet in thickness, have been reported within the buried valley beneath Cooking Lake.

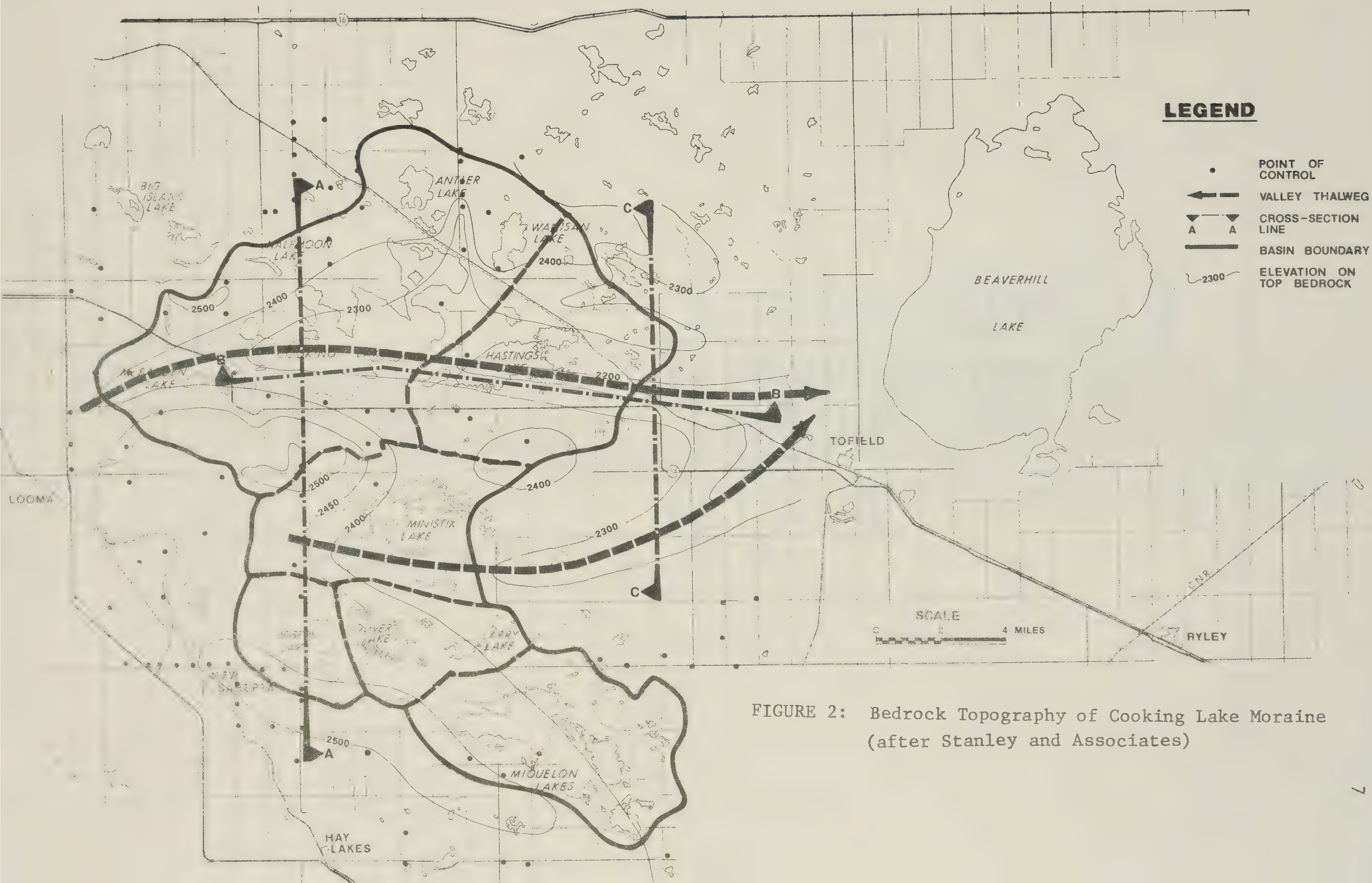
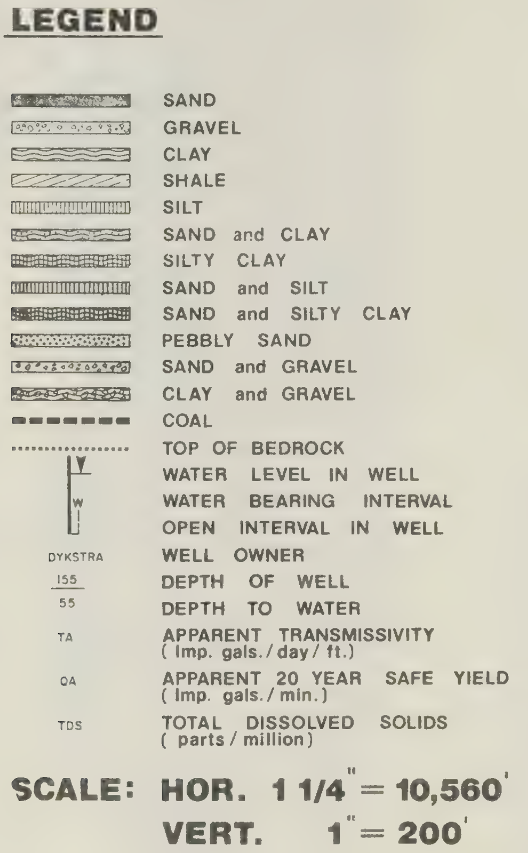
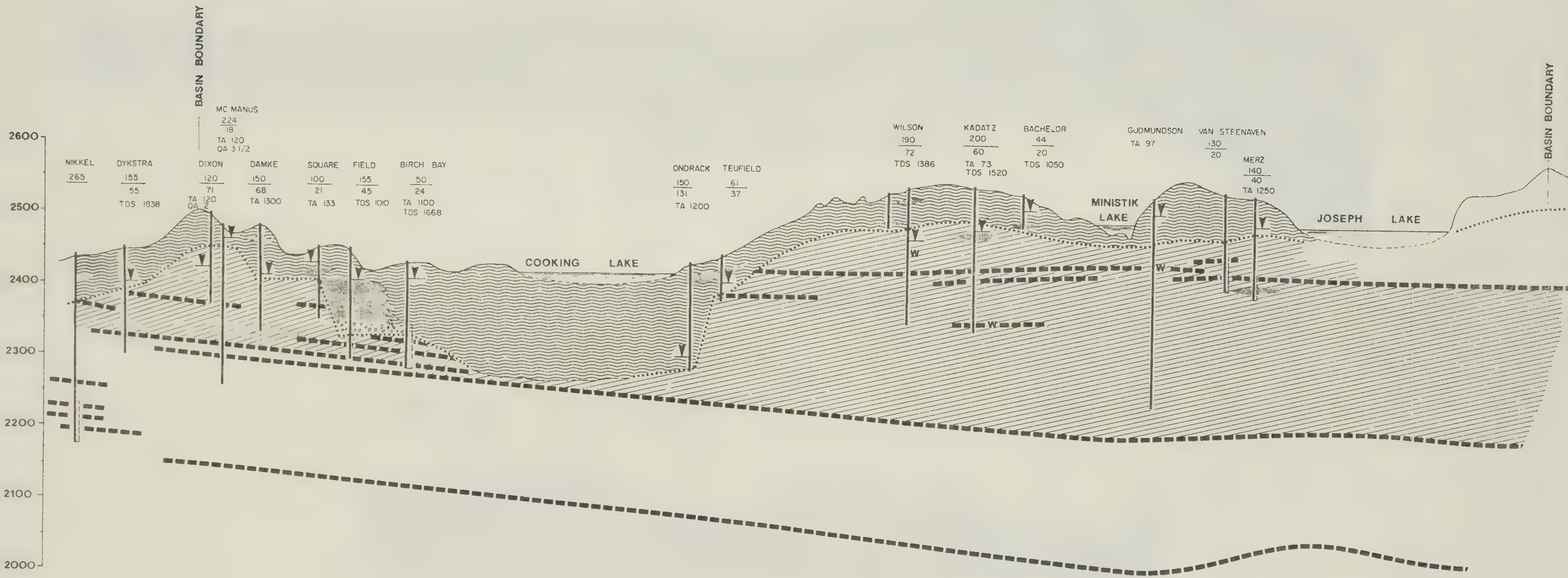


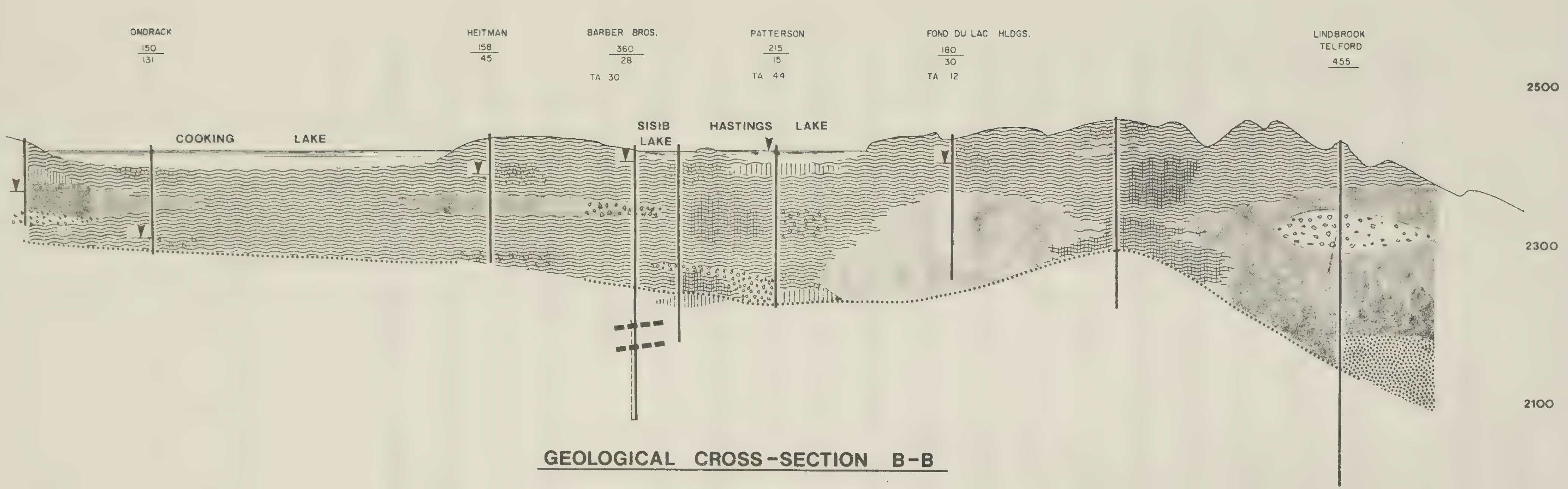
FIGURE 2: Bedrock Topography of Cooking Lake Moraine (after Stanley and Associates)

FIGURE 3: Geological Cross-sections (after Stanley and Associates)





GEOLOGICAL CROSS-SECTION A-A



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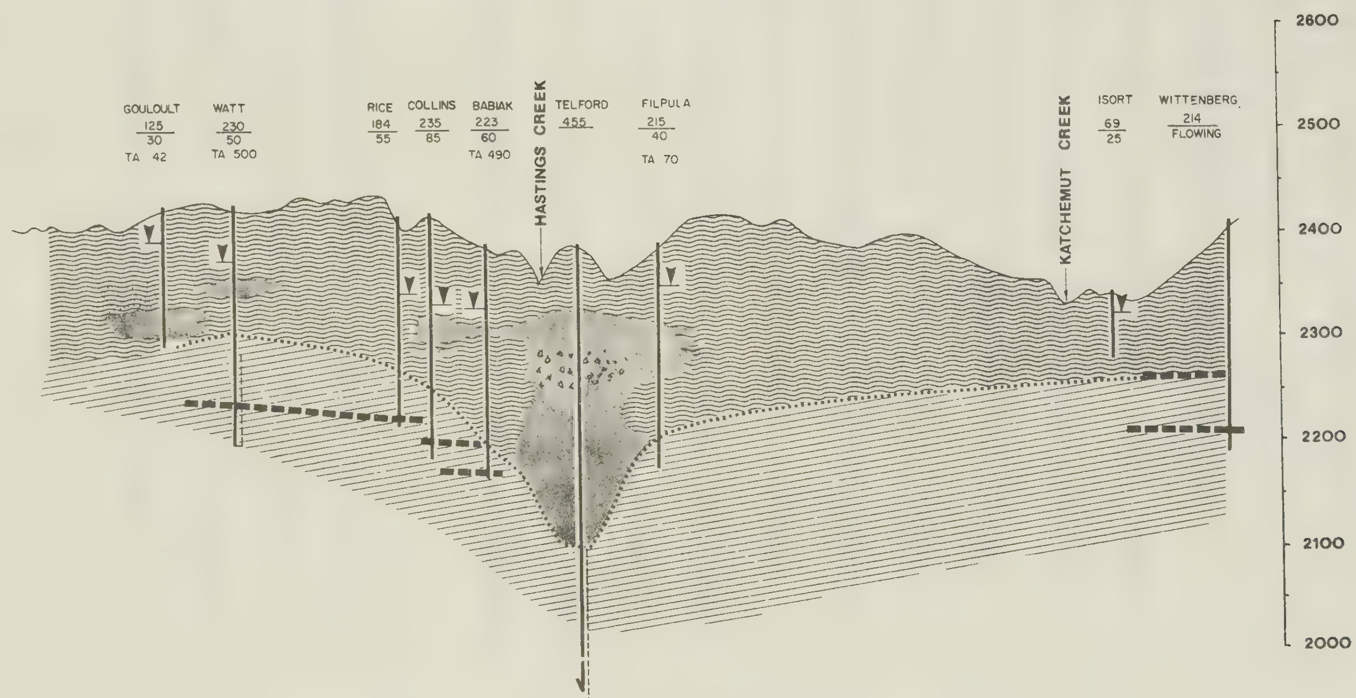


GEOLOGICAL CROSS-SECTION C-C

CHAPTER 2

WATER BALANCE MODEL

Introduction

The Cooking Lake Moraine contains a system of interconnecting lakes which can, depending upon the lake levels, contribute to the next downstream neighbour. A primary flow series can be seen from Miquelon Lake through Larry Lake to Oliver Lake. Oliver Lake to Ministik Lake to Cooking Lake and finally to Hastings Lake will complete the primary series (Figure 4). The flow continues from Hastings Lake to Beaverhill Lake which discharges to the North Saskatchewan River. Primary Lakes are defined as those which are on the main flow system, receive a contribution from another lake and contribute to other lakes. Secondary Lakes are those which only contribute to the water balance of other lakes. The secondary system consists of Joseph Lake, Half-Moon and Antler Lakes, and Wanison Lake. Figure 4 shows a cross-sectional profile of the Cooking Lake Moraine with a few reference elevation lines, and Figure 5 is a schematical flow chart illustrating the idea of primary and secondary flow systems.

The main components of any regional water balance model are shown in Figure 6. The interdependence of these components can be seen by the inspection of the following equation (Eqn. 1) which mathematically defined the water balance relationship with respect to lake level fluctuations.

$$\Delta S = PPT + IF \pm GW + R - OF - E \qquad (1)$$

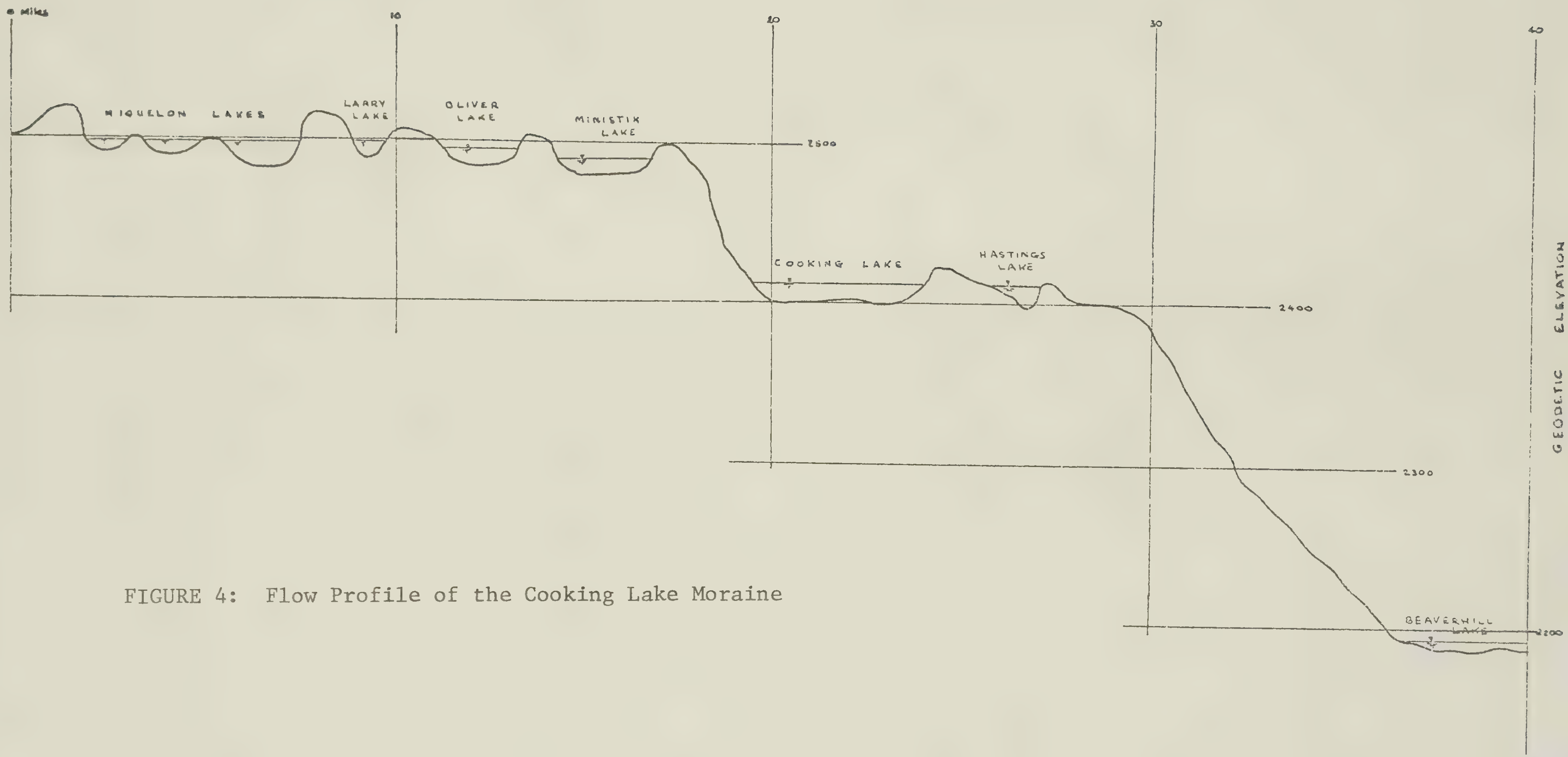


FIGURE 4: Flow Profile of the Cooking Lake Moraine

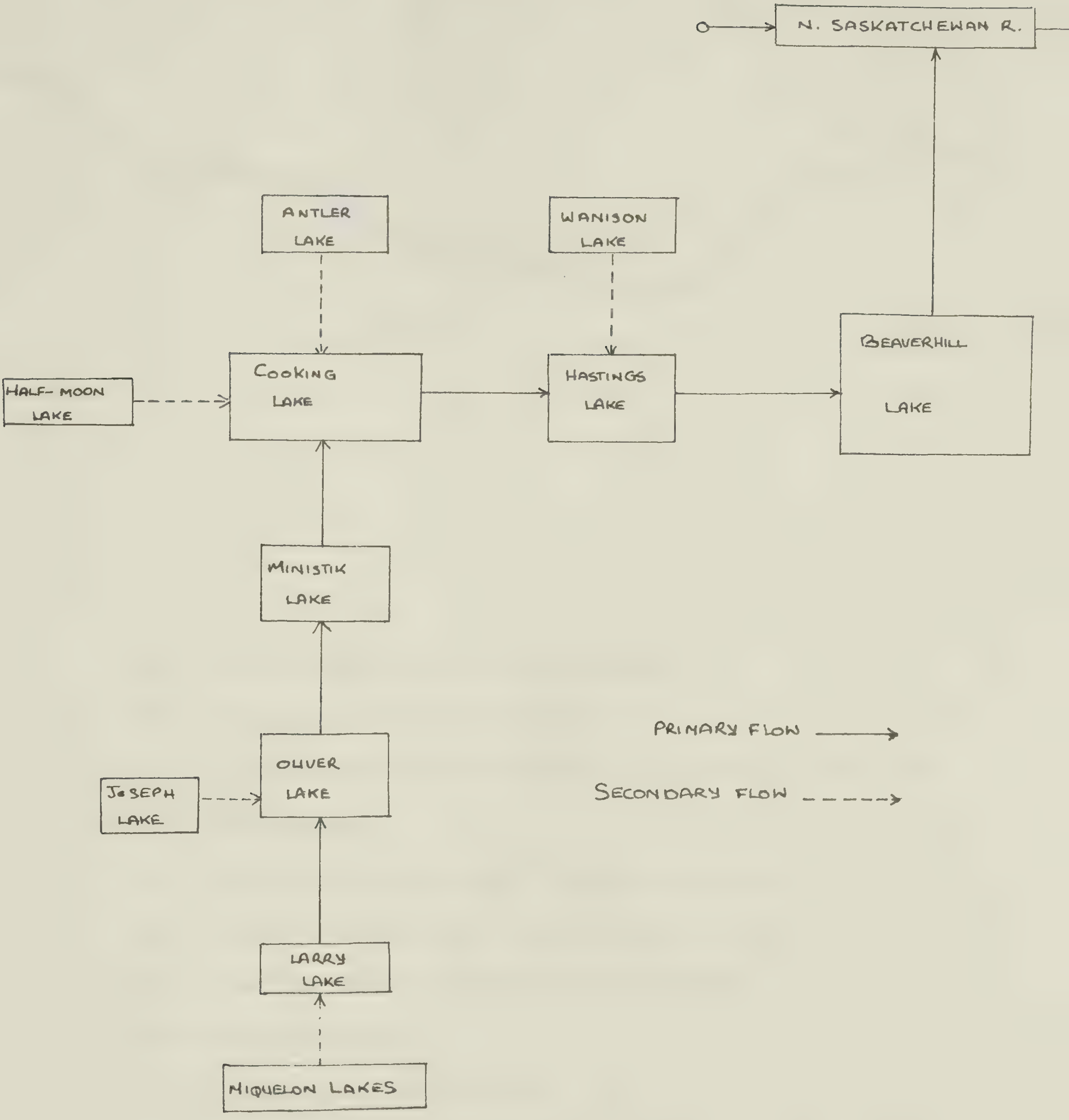
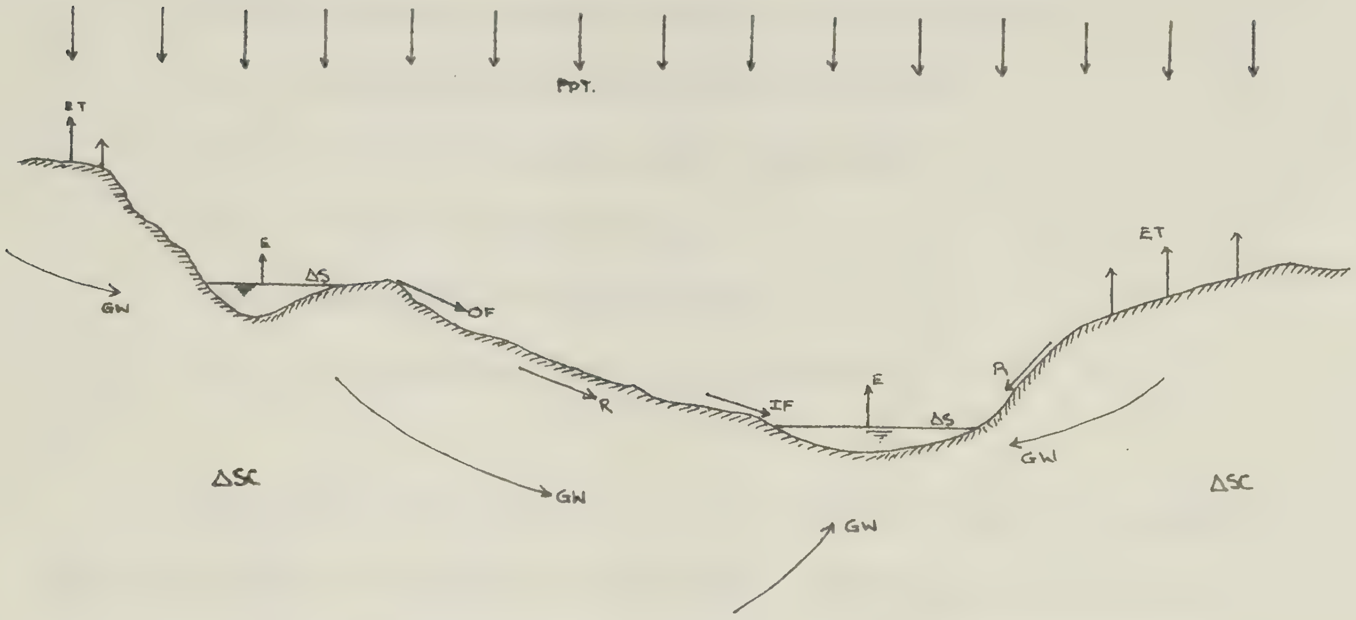


FIGURE 5: Flow Chart of Lake Dependence

FIGURE 6

REPRESENTATION OF WATER-BALANCE EQUATION COMPONENTS



ET : Evapotranspiration of land surfaces

E : Evaporation from lake surfaces

R : Runoff from land after storage capacity of ground has been filled

GW : Ground water input into the lake watershed

ΔS : Change in lake level or water stored in lake

OF : Outflow from lake to the next watershed

PPT: Precipitation

IF : Inflow into the watershed

ΔSC: Change in ground storage

where:

ΔS = change in lake storage

PPT = precipitation on lake and land surfaces

IF = inflow from streams and other lakes

R = runoff from the lake's watershed

OF = outflow from the lake

GW = groundwater contribution

E = evaporation from lake surface

A convenient time interval is usually selected and is dependent upon the interval at which the independent variables of equation (1) are measured. All of the above components will vary with the time of year and with the physical and climatical conditions prevailing at any particular time. Each component in turn is considered to determine its significance with respect to the lake level fluctuations.

Precipitation

As in many hydrological studies the lack of suitable measurements will hinder the development of a model and will necessitate the usage of basic simplifying assumptions, which may or may not be valid. Precipitation records for the immediate Cooking Lake area are unavailable for the length of period required. It is therefore necessary to adopt records of neighbouring stations. Edmonton and Edmonton International Airport are two such stations situated approximately twenty miles north-west and south-west respectively, from Cooking Lake. The record periods are, 1880 to present for Edmonton and 1960 to present for Edmonton International Airport.

Numerous investigators of hydrological phenomena have noted that, under particular conditions, naturally occurring variables such as precipitation and temperature conform to a normal frequency distribution. [Markovic, 1965. Yevjevich, 1972]. The normal distribution was probably first described by Abraham De Moivre in 1733 and used later by Gauss to describe the theory of accidental errors of measurements involved in the calculation of orbits of heavenly bodies. For the normal distribution only two parameters, the variance and the expected value, are needed to completely describe the distribution.

The remainder of this section will be devoted to the determination of the frequency distribution associated with total monthly precipitation. When this distribution is known a sample of some length (say 500 years) will be generated by the experimental statistical technique (Monte Carlo Method).

The normal distribution has been observed to be followed by many hydrological variables whenever these variables are taken in their natural or in some transformed state. It is therefore obvious to start an investigation of this type by testing the normalcy of the available time series. The theoretical fitting of the normal distribution to an empirical distribution often causes difficulties. One of the main problems is associated with the property of the theoretical normal variate being able to assume any value between plus and minus infinity, whereas the many hydrological variables measured, assumed to be normally distributed, are positive valued with a lower boundary, usually zero. Moreover if the properties of the normal distribution are recalled, the probability of the variate being negative or reaching the lower boundary is very small if the mean is greater than three standard deviations.

In many cases this minute probability can be neglected and the bounded, positive valued variable can be assumed to be normal.

Several precedures are currently in mode to show the applicability of the normal distribution to an observed time series. The simplest is a frequency plot of the magnitude of the variable against a plotting position, usually equal to the rank of that value divided by N + 1, where N equals the total sample length. The graphical method for the determination of the parameters of a distribution is limited in its accuracy as several investigators will produce different results, but it does show how plausible the normal fit will be. The plots of total annual precipitation, natural log and cube root of total annual precipitation, mean and natural log of mean annual temperature are given in appendix 3 as an example of the type of plots produced. The best procedure, to standardise results, is to use a mathematical approach to obtain the sample's distribution parameters.

There are basically four methods of estimating the population parameters of a distribution function from a sample of that population. Generally these are known as, the least squares method, the method of moments, the maximum likelihood method and the graphical method described earlier. The first three effectively give the mean and variance of the normal function to the same accuracy.

The population mean is estimated by the sample mean, $\bar{x}$, and the population variance by the unbiased sample variance, $\hat{S}^2$. For the case of a discrete variable, the mean is given by

$$\bar{x} = \frac{1}{N}\sum_{j=1}^{N} x_j \tag{2}$$

and the variance by,

$$\text{unbiased,} \quad \hat{S}^2 = \frac{1}{N-1} \sum_{j=1}^{N} (x_j - \bar{x})^2 \tag{3}$$

where,

N = number of observations

x_j = jth observation of variable x.

and other terms defined above

In terms of moments; equation (2) defines the first moment about the origin and equation (3) defines the second central moment about the mean with a correction for bias. Using equations (2) and (3) the population estimates of expected value and unbiased variance for total monthly precipitation were obtained. These estimates are given in table 1. Also given in table 1 are the estimates of the population's skewness and kurtosis coefficients. The skewness coefficient can be derived directly from third and second central moments and the coefficient of kurtosis from the fourth and the second central moments. Mathematically the definition of skewness is given by equation (4) and kurtosis by equation (5). The Excess Coefficient is another useful distribution parameter and is defined by equation (6)

$$\hat{C}_s = \frac{N}{N-1} \frac{M_3}{M_2^{3/2}} \tag{4}$$

$$\hat{C}_k = \frac{N^3}{(N-1)(N-2)(N-3)} \frac{M_4}{M_2^{2}} \tag{5}$$

$$\hat{E} = \hat{C}_k - 3 \tag{6}$$

where

$\hat{C}_s$ = unbiased estimate of the population skewness coefficient

$\hat{C}_k$ = unbiased estimate of the population coefficient of kurtosis

$\hat{E}$ = unbiased estimate of the population excess coefficient

and

M_i = ith central moment

$$= \frac{1}{N} \sum_{j=1}^{N} (x_j - \bar{x})^i$$

The coefficient of skewness is a measure of the asymmetry independent of the dimension of the variable and the coefficient of kurtosis measures peakedness or flatness. These two estimates of the population parameters provide useful tests to the validity of assumming normalcy of a time series. This is due to the normal distribution having a coefficient of skewness equal to zero and the coefficient of kurtosis equal to three or the excess coefficient equal to zero.

The following two identities have been used to test for Normalcy. (Yevjevich, 1972).

(1) Unbiased skewness coefficient, $\hat{C}_s$ satisfies either

(a) $$-0.10 \leq \hat{C}_s \leq 0.10 \tag{7}$$

or more confining

(b) $$-0.05 \leq \hat{C}_s \leq 0.05 \tag{8}$$

and, (2) Unbiased Excess Coefficient

$$-0.5 \leq \hat{E} \leq 0.5 \tag{9}$$

TABLE 1

Unbiased population estimates of mean, variance, coefficient of skewness and coefficient of kurtosis for total monthly precipitation measured in inches at Edmonton (Industrial Airport).*

Edmonton N = 91

Month	Mean	$\hat{S}$	$\hat{C}_s$	$\hat{C}_k$
JAN	0.93	0.54	0.70	3.44
FEB	0.71	0.53	1.13	4.34
MAR	0.77	0.58	1.53	6.76
APR	0.93	0.67	1.41	5.70
MAY	1.76	1.15	1.87	10.55
JUNE	3.14	1.58	0.79	3.95
JULY	3.33	1.59	1.90	9.61
AUG	2.50	1.42	0.54	2.97
SEPT	1.35	0.93	1.00	3.99
OCT	0.74	0.56	0.67	2.57
NOV	0.74	0.56	1.57	7.10
DEC	0.83	0.57	1.40	7.10
ANNUAL	17.72	3.66	0.32	4.06

* Monthly Records - Atmospheric Environment Service

For the ninety-one years of records recorded at Edmonton, estimates of both the skewness and kurtosis coefficients have been obtained and the identities given by equations 8 and 9 applied. The conclusion drawn from these two tests is that the variable, total monthly precipitation, does not have an empirical distribution function that is close to the normal distribution function. Fortunately there are two alternatives still available. The first alternative is to try and fit some other theoretical distribution to the observations and the second is to transform the variable so as to obtain a 'new' variable which may be normally distributed.

The second alternative is the more obvious and convenient path to follow at this stage. A transformation will still allow the use of the simple properties of the normal distribution and prevents the use of a theoretical distribution function which may give a close fit but would be difficult to justify.

Transformations to observed data take many forms, the most common being the natural logarithm. This transformation proved to be not normally distributed, that is, not log-normal, when the procedure and tests described earlier in this text were applied. The next most common transformation usually takes the form described by equation (10).

$$T(x) = X^{\pm I} \qquad (10)$$

where:

X = Random variable

$T(X)$ = transformed variable

I = transforming agent.

For the time series, given by the records of total monthly precipitation, a series of I values were applied and the distribution parameters of, expected value, variance, $\hat{C}_s$, $\hat{C}_K$ obtained. The values of $\hat{C}_s$ and $\hat{C}_K$ are tabulated in table 2 for each I. The column titled 'closest to normal' in table 2 shows the transformation that give values of $\hat{C}_s$ and $\hat{E}$ closest to meeting the criteria set by equations 7 and 9. This shows that several transformations can be applied to the data but it would appear that the square-root is the most applicable. The square-root gives the value of $\hat{C}_s$ and $\hat{C}_k$ closest to normal whenever the square-root and another transformation lie inside the limits. On a few occasions the values of $\hat{C}_s$ and $\hat{C}_k$ lie just outside the limits of equations 7 and 9. The limits seem to be between those defined by equations 11 and 12 which are not all that different for the normal condition.

$$(1) \quad -0.3 \leq \hat{C}_s \leq 0.3 \tag{11}$$

$$(2) \quad -1.0 \leq \hat{C}_k \leq 1.0 \tag{12}$$

the values of $\hat{C}_s$ and $\hat{C}_k$ all fall within these new limits for the same transforming agent except for the month of July where $\hat{C}_k$ is still too high. Table 3 lists the month and the suitable transforming agent.

TABLE 2

Unbiased Population Estmiates of $\hat{C}_s$ and $\hat{C}_k$ for tranformed Monthly Total Precipitation Measured in ins.

At Edmonton

Transformed Variable = $(\text{Precipitation})^I$, where I = Transforming Agent

	Coefficient of Skewness $\hat{C}_s$ Transforming Agent						Coefficient of Kurtosis $\hat{C}_K$ Transforming Agent						Closest To Normal	
MONTH	0.25	0.33	0.4	0.5	1.0	1.5	0.25	0.33	0.4	0.5	1.0	1.5	$\hat{C}_s$	$\hat{C}_k$
JAN	-1.686	-0.79	0.617	-0.266	0.70	1.377	9.747	5.67	4.30	3.406	3.44	5.163	0.5	0.5
FEB	-1.591	-0.88	-0.487	-0.058	1.13	1.901	7.474	4.90	3.918	3.302	4.34	6.955	0.5	0.5
MAR	-0.854	-0.21	0.098	0.412	1.53	2.764	6.836	4.19	3.538	3.402	6.76	14.353	0.4	0.5
APR	-0.521	-0.24	-0.036	0.249	1.41	2.339	4.011	3.64	3.497	3.501	5.70	9.764	0.4	0.4
MAY	-0.376	-0.16	0.027	0.304	1.87	3.729	3.792	3.81	3.939	4.331	10.55	24.474	0.4	0.4
JUN	-0.259	-0.14	-0.044	0.097	0.79	1.488	2.998	2.94	2.921	2.949	3.95	6.329	0.4	0.4
JUL	-0.400	-0.03	0.236	0.582	1.90	3.082	7.629	6.65	6.223	6.046	9.61	17.288	0.33	0.5
AUG	-0.880	-0.63	-0.458	-0.237	0.54	1.134	4.585	3.79	4.356	2.955	2.97	4.423	0.5	0.5
SEP	-1.627	-0.92	-0.539	-0.130	1.00	1.729	8.161	5.20	4.102	3.381	3.99	6.426	0.5	0.5
OCT	-0.784	-0.34	-0.137	0.069	0.67	1.147	4.767	2.99	2.467	2.181	2.57	3.848	0.5	0.33
NOV	-1.367	-0.71	-0.342	0.092	1.57	2.856	6.970	4.65	3.927	3.668	7.10	14.167	0.5	0.5
DEC	-1.739	-0.96	-0.544	-0.101	1.40	2.852	9.178	5.70	4.510	3.908	7.18	15.412	0.5	0.5
YEAR	-0.323	-0.25	-0.189	-0.101	0.32	0.736	4.169	4.09	4.033	3.976	4.06	4.682	0.5	0.5

TABLE 3

DETERMINED TRANSFORMING AGENTS FOR PRECIPITATION

Month	Transforming Agent
JAN	0.5
FEB	0.5
MAR	0.4
APR	0.4
MAY	0.4
JUNE	0.4
JULY	0.33
AUG	0.5
SEPT	0.5
OCT	0.5
NOV	0.5
DEC	0.5
YEAR	0.5

Inflow Parameter

Inflow to a lake sub-system can take one, or a combination, of the following basic forms; surface runoff, groundwater (which can appear above or below the lake surface) and streamflow, primarily from one lake to another. Surface runoff and groundwater components of the inflow parameter will be dealt with in later sections as they warrant separate treatment.

The schematic representation of the lake system shown in Figure 5 illustrates the path water would take if each lake were full and discharging its surplus water to the next lake in the series. The stage at which the lakes begin to discharge is controlled by the level of the overflow channel. This level can only be estimated roughly, at this moment, from topographical maps of each lake. These estimates are given in table 4. The water balance model should have a routine incorporated into it to estimate, for a given time period, the amount of water a lake will contribute to the water balance of another lake. Losses will naturally occur in the channel connecting the lakes, particularly after a dry, no flow period. These losses can be attributed to the moisture deficit that occurs in any natural channel if it is left dry, to evaporation from the water surface and transpiration of phreatophytes and other plants growing in the channel. The practical estimation of these losses is impossible due to the number of affecting variables such as channel dimensions, materials composing the channel sides and bed, time of year, frequency and magnitude of discharges through the channel and the type and state of plant growth. Moreover, the magnitude of these intermittent losses is negligible when compared to the inaccuracies

of measuring the other water balance components,in particular surface runoff.

By considering the event that a lake has an excess to discharge it is justifiable to assume that the magnitude of water transferred from one lake to another is equal to the excess of the contributing lake calculated for that time period.

TABLE 4

MAXIMUM AND POSSIBLE MINIMUM LAKE LEVELS ABOVE ORDINANCE DATUM.

(OBTAINED FROM 1:25000 TOPOGRAPHICAL MAPS).

LAKE	MAXIMUM FT.	LEVEL M	MINIMUM FT.	LEVEL M
Miquelon	2515	766	2483	756
Oliver	2505	763	2485	757
Joseph		761.5		755
Ministik	2500	761.4	2480	755
Cooking	2423	738	2400	731
Hastings	2414	735	2397	730

Groundwater

The flow of groundwater into and over the subsurface boundaries of the Cooking Lake Moraine in governed mainly by the topography of the area. To describe in detail the numerous groundwater flow patterns that could exist in such a complex situation as is to be found in the Moraine would require an extensive well monitoring program and is beyond the scope of this study. The Moraine is particularly complex due to the

diversity in surface and sub-surface materials and to the complicating aspects of permanent lakes on any groundwater system. The Moraine is a local topographical high and as such can be generalised to the typical prairie profile (Meyboom),1962, 1966), but it must be said that the local groundwater inflow and outflow of individual lakes is at the present time an unknown factor in the water balance of each lake.

The prairie profile has been offered as a model of the typical prairie groundwater flow condition to which all observable groundwater phenomena can be related. The basic properties of the model are shown in Figure 7. Geologically the profile ideally consists of two layers, the uppermost being the least permeable, with a steady-state flow of water to a discharge area. The prairie profile can often be considered as a model for small scale systems, such as the typical knob and adjacent kettle, common to rolling prairie topography~~s~~ and as a model for wider, regional scale systems. The applicability of the profile has been substantiated by numerous borings for both scale systems.

It is often quite feasible to delineate between recharge and discharge areas because each has particular and distinctive characteristics, the most prominent being the change in potential with increasing depth. An increase in potential with depth indicates a discharge area and a decrease in potential with depth a recharge area. Flowing wells are found in discharge areas in many instances.

When groundwater is discharged from the groundwater system to a surface system it is usually lost from the entire water system by evapotranspiration. Considerable attention has been given to evapotranspiration and to the many surface features that are associated with it. The estimation of evapotranspiration, in particular, will be discussed in the next Chapter. The remainder of this section will concen-

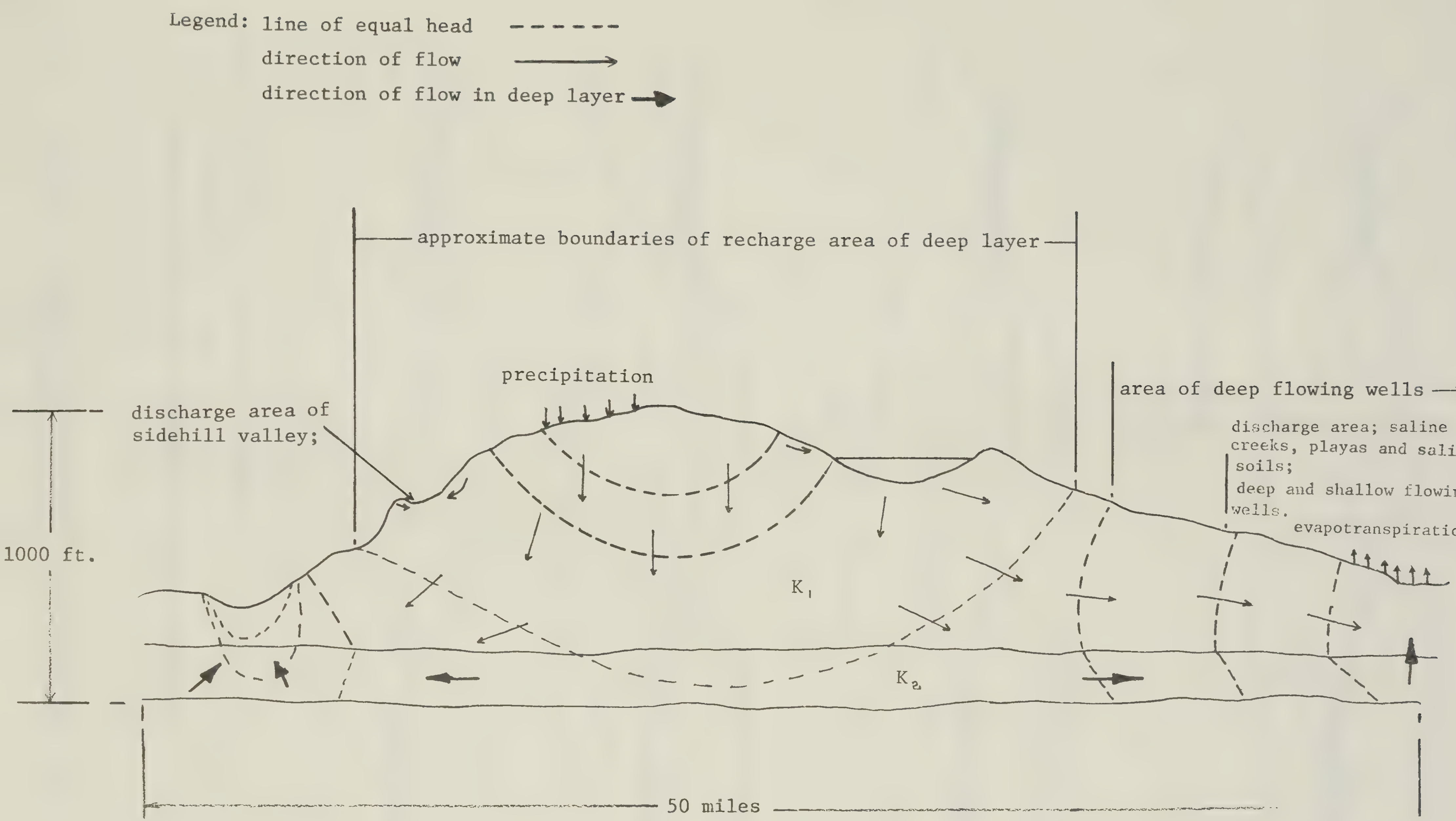
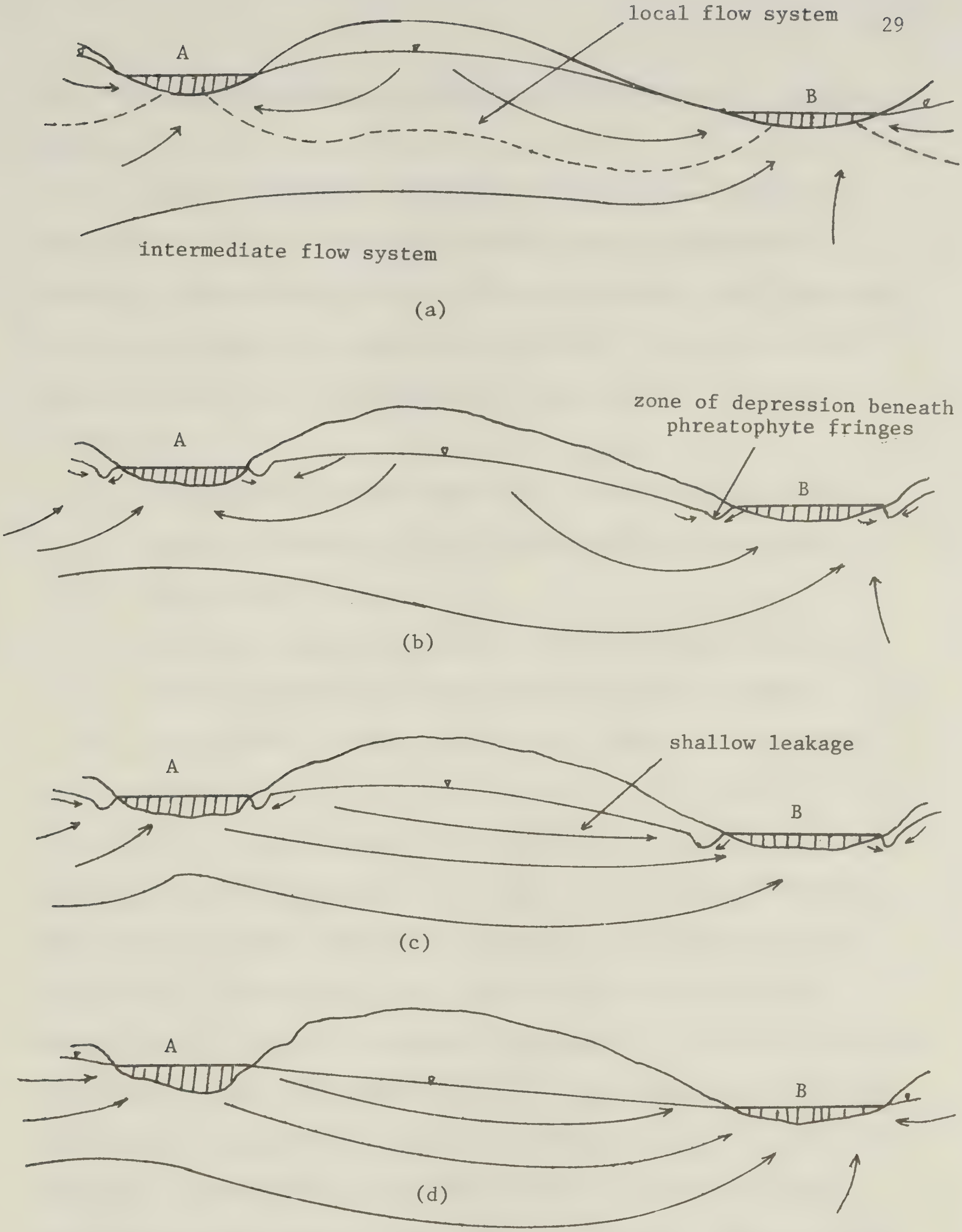


FIGURE 7: The Prairie Profile (after Meyboom, 1966)



Legend: Direction of groundwater movement ——→

Watertable ——▽——

Boundary between flow systems of different magnitude - - - - - -

FIGURE 8: Diagrams of Flow Conditions Near Permanent Lakes
(after Meyboom, 1966)

trate on the surface features associated with discharge and recharge areas.

The spatial investigation of various vegetal types, within a given watershed, will often yield valuable clues to the physical character of groundwater flow and will directly give a good indication of soil type (hence for runoff characteristics). The usual observations associated with discharge and recharge areas include the following,

(a) the occurrence or non-occurrence of willow rings and the chemical character of water bodies centered within them when they do occur.

(b) The distribution of halophytic communities with respect to the occurrence of local and regional flow systems.

(c) The location of ponds and bogs with respect to groundwater flow.

The occurrence of willow rings centered around higher areas, that is, recharge areas, indicates local discharge points within the recharge area, [Meyboom, 1966]. Each discharge point is characterised by a willow ring. Willows have a very low tolerance for saline soils, indicating their affinity for waters which have not travelled far in the groundwater system. The typical basin will have numerous willow rings in its' higher areas, wheras the lower regions are void of rings. The occurrence of willow rings in an indication of a local flow system superimposed on the regional groundwater flow system. [Meyboom, 1966].

Whenever there is a nett upward movement of mineralised groundwater, extensive saline soil areas occur. In many cases the transitional changes of groundwater and vegetation within a basin are from relatively

pure water and willow rings in the recharge areas to saline water and halophytic plants in the discharge areas. When the local flow system is not replenished by saline water formations, the groundwater is relatively fresh and saline soils fail to develop. However if there is still a net surplus of water the result will be the occupation of the area by fresh water phreatophytes. Several of the above features can be found in the Cooking Lake Moraine.

In areas of hummocky terrain (see description of the surficial geology, Chapter 1) ephemeral water bodies have been found to function as recharge points during spring and early summer, and as discharge points during summer and autumn. Permanent lakes however have been shown to be areas of permanent groundwater discharge. [Meyboom, 1966, 1967]. The four typical flow conditions near permanent lakes are shown in Figure 8 a-d. These four conditions are believed to be similar to the groundwater flow conditions found around the lakes in Cooking Lake Moraine.

The first situation, figure 8a, is a spring condition where discharge from local and the intermediate flow system moves towards the lakes. Figure 8b is an approximate summer condition of seepage towards the phreatophyte fringe surrounding the lake. Insufficient recharge causes a deterioration of local flow and will result in shallow movement of groundwater from Lake A to Lake B (Figure 8c). Figure 8d shows the final deteriorated system for the autumn and winter condition.

The numerous studies made of the groundwater flow condition near lakes demonstrates the dynamic nature of lake bodies and the groundwater flow condition. This implies the virtual impossibility of describing the nature of groundwater movement, in the vicinity of lakes, in terms

of static analysis. In many cases it is safe to assume that there are sub-surface flow into the lake from the watershed and sub-surface flows between the lakes from the watershed. The quantities of water involved in each sub-surface flow system is believed to be negligible in comparison with the surface runoff and evaporation components of the water balance of the area. A low quantity of groundwater flow between the lakes can be seen to be feasible if the geological cross-sections defined in Figure 2 and shown in Figure 3 are inspected. In Figure 3 it can be seen that clay (an aquiclude) has extensively covered the area and overlies another low permeability material, shale.

Springs, above and below the surface of Miquelon Lake have been reported [Nyland, 1970] and Duck's Unlimited mentions the presence of springs at Ministik Lake in the 1930's. It has been hypothesised that springs must be an important feature in the hydrology of Hastings Lake if this hydrology is to adequately explained. [Kerekes, 1965]. All of the above observations point towards the complexity of the situation to be found in the Moraine. Therefore to model groundwater flows properly the geology of the area must be investigated in greater detail than is possible here. The groundwater condition is not a static situation in the case of the Cooking Lake Moraine and so a 'lump' type system of modelling will not accurately reflect the stochastic variations within the groundwater system. Consequently the groundwater contributions to the water balance must be ignored.

<u>Surface Runoff</u>

In very general terms the water yield of a basin (which includes surface runoff and groundwater flow) is simply the difference between

the precipitation over the total watershed (land surface and open water surfaces) and the losses due to evaporation and transpiration. The combined loss due to evaporation from open water surfaces and the ground and transpiration by vegetation is an important component of the water balance equation which is difficult to quantify.

Many techniques have been developed to estimate evapotranspiration. Some techniques rely on elaborate theories, such as the aerodynamic approach or eddy-correlation theory, others are based on these theories but have been empirically derived. Because of the importance of evaporation and transpiration in any water balance model a brief overview of the main estimation techniques is given in Chapter 3.

Summary of Chapter

If the losses due to evaporation and evapotranspiration are known and the amount of precipitation that has fallen over the watershed is measured, for the same time period, it is possible to give a value for the expected water yield. The spatial variation of precipitation, in particular, over any area is one of the most variable parameters in the water balance of that area. As the spatial measurement of precipitation over any appreciable area is inpracticable it is necessary to adopt point measurements as a representation of spatial precipitation amounts. The problems encountered and the errors associated with adopting point measurements as a model of regional precipitation has been the topic of many papers and discussions. [Ferguson and St rr in 1967].

Groundwater contributions to the water balance of the Cooking Lake Moraine has been shown to be a dynamic problem with a considerable variation between the seasons. These contributions however, are not

as important as the surface runoff or precipitation components of the water balance so the assumption of negligible groundwater input or output from the water system is acceptable.

Like precipitation, evapotranspiration estimation is limited by the extend and practicability of the variables measured, the type of variable measured and the reliability of the measuring techniques. In the long run the estimation technique adopted to give a measure of surface runoff is determined by the availability of information. Information that is available consists of records of precipitation (total daily at Edmonton), mean daily temperature (maximum, and minimum) for a relatively long period, that is, greater than forty years. Information such as maximum and minimum lake levels are obviously limited in accuracy because of the means by which they were obtained.

CHAPTER 3

ESTIMATION OF EVAPORATION AND EVAPOTRANSPIRATION

This Chapter deals with the estimation of open water evaporation and evapotranspiration from land surfaces. It is necessary to deal with this topic in the detail given here because of its' relative importance in the hydrology of any area and it is especially important when investigating a lake district such as the Cooking Lake Moraine. Some of the methods outlined are only suitable for the more elaborate or extensive water balance studies. Consequently these methods are only of academic interest and have been included in this dissertation only for completeness.

The selection of a particular method or empirical formula is based primarily on the degree to which the area under study has been monitored. Whether or not the various independent variables of each method have been measured and are available will determine the selection of the technique incorporated into the model to measure the evaporation or evapotranspiration component.

There are several methods of estimating evaporation and transpiration, the accuracy of each method varying considerably from one location to the next and with respect to eachother. The more commonly used methods include [Munn, 1961].

(a) The Water budget (or storage equation)

(b) The Energy budget

(c) The Eddy-Correlation method

(d) The Aerodynamic-profile method (humidity and wind velocity gradients).

Combinations of the above four methods form the basis of many of the empirical formulae which have been derived and are in use today. The more popular of these formula are those which have been derived by the following experts and the formula's usually carry their name

(1) Penman; (2) Thornthwaite and (3) Turc

Many other formula have been derived by combining properties of the above estimation methods. The more recent formula's tend to improve the results when compared with the results of classical formulas but are usually only applicable to the conditions found at one locality. This type of local formula include those of, Blaney and Criddle, McIlroy, Rohwer and Meyer and Tichomirov.

Methods Used to Estimate Evaporation

There are several formal approaches used in the estimation of evaporation and transpiration, two of which are equally applicable to evaporation from a free water surface and evapotranspiration from vegetation. These two are the Water-budget method and the Eddy-correlation method.

Water-Budget Method

This method generally involves the use of a storage equation of the form given by equation (14)

$$P + I \pm u = E + 0 \pm \Delta S \tag{14}$$

where

P = total precipitation falling on the area

I = volume of inflow

u = volume of underground inflow or outflow

E = evaporation, or evapotranspiration from the surface of the area

0 = outflow volume

ΔS = change in storage (in reservoir or groundwater)

The water-budget method can be applied both to evaporation and evapotranspiration but the estimation of the latter is more difficult than that of the former. The main reason for this is the difficulty in estimating the change in storage, particularly groundwater storage changes, and losses to deep percolation. Changes in water-table level can be easily monitored but translation of such fluctuations into an actual change in storage is difficult.

The units of equation (14) are usually inches or millimetres over a given area for some convenient time interval. One main disadvantage of this method is that each variable can only be measured to a particular degree of accuracy and accumulation of these errors in equation (14) would result in poor estimates of evaporation. To illustrate this point considerthe F.D. Roosevelt Lake, formed by the Grand Coulee Dam, on the Columbia River. The annual evaporation of this lake has been estimated at forty-two inches, or equivalent to 290,000 acre-feet. If the reservoir outflow (estimated at about 70 million acre-feet) was measured to an accuracy of 5%, the error would be about ± 3.5 million acre-feet or more than ten times the total evaporation. It can be seen that for certain large catchments or large lakes, an accurate water-budget approach to estimating evaporation or evapotranspiration is not practical. Many safeguards are necessary to ensure that all variables are measured to a

realistic degree of accuracy. If this accuracy could be achieved the water-budget method would be ideally suitable for evaporation estimation particularly for the smaller watersheds.

Energy-Budget Method:

The energy-budget or heat-budget was first used by Schmidt in 1915 to compute evaporation from oceans. Many other investigators have attempted to use the technique to compute evaporation from bodies of water of all sizes and to estimate evapotranspiration. Only in the last decade has the necessary instrumentation become available to permit the measurment of certain variables in the technique.

The energy-budget method is based on the principle of the conservation of heat-energy within a body of water. This conservation principle states that a balance must exist between; (a) insolation, (b) heat transferred from the water surface by radiation, convection and conduction, (c) heat energy acquired or lost in raising or lowering the temperature of the water, and (d) heat dissipated or acquired by evaporation or condensation. If items (a), (b) and (c) can be measured, evaporation or condensation may be estimated.

The water-budget was used by Anderson in 1950-1951 as a control to test the energy-budget method for estimating evaporation from Lake Hefner, Oklahoma (Anderson, E.R., 1954). He concluded that the method gave satisfactory results for periods of ten days or more. Unacceptable results were obtained for periods less than 10 days for Lake Hefner. Generally periods of one or two weeks have been used for most energy-budget estimates.

The use of the energy-budget equation to estimate evapotranspiration is not applicable because the evaporating or transpiring surface is three dimensional. For evapotranspiration measurements using an energy-budget appraoch a more complex energy relationship is required. Assumptions to the extent that energy losses due to precipitation, Snowmelt and runoff are negligible and that energy storage is minimal, are required if a workable equation is to be found. The type of equation derived is similar to equation 15.

$$E = \frac{R_n - S}{(1 + \beta)} \tag{15}$$

where:

E = evaporation

R_n = net radiation flux

S = soil heat flux

β = Bowen's ratio, which can be defined as the ratio of the loss of upward energy flux as sensible heat to the energy flux used in evaporation.

To reach reliable conclusions from energy-budget studies, detailed measurements under different weather conditions are required, preferably continuous records of the various variables in the energy balance relationship.

Eddy-Correlation Method:

This method, also known as the eddy-flux or eddy-transfer method, is equally applicable to evaporation from a free water surface or to

evapotranspiration. It employs the measurement, at some arbitary point, of vertical turbulent fluxes in the atmosphere of both velocity and water vapour. The basic principles of eddy-correlation were first recognised in 1951 (Swinbank, 1951).

From a theoretical point of view, this method is quite straightforward. However, the measurement of simultaneous fluctuations of the deviations from the mean of the vertical wind velocity and specific humidity, requires an instrument sensitive enough to measure these rapid changes and sturdy and stable enough for continuous use. At the present time no instrument has been developed that is capable of measuring only the eddy-flux. Instruments do exist however that do measure total-flux and eddy-flux combined with the flux due to mean motion.

Aerodynamic-Profile Approach:

The two premises below are the base of this method [Sutton, 1953]

(a) If a moisture gradient exists in the air, water vapour will move towards points of lower moisture content.

(b) The rate of movement of the water vapour is accentuated by the intensity of turbulence in the air.

The profile technique concerns itself with the turbulent transfer of water-vapour between two levels in the air at a small distance above the evaporating surface. It is usually assummed that:

(1) Diffusion in the forward direction is negligible in comparison with forward transportation by the wind.

(2) Steady-state conditions exist, and

(3) There is an infinite line source of water, that is, the problem is two-dimensional.

An equation can be derived by analogy with molecular and other well known transfer processes. Accepting assumptions (1), (2) and (3) are valid and that there are spherical eddies with a logarithmic wind profile above the evaporating surface, an equation of the form given by equation 16 can be derived. This is in effect the classical aero-dynamic-profile equation derived by Thornthwaite and Holtzman. (Thornthwaite and Holtzman, 1939)

$$E = \frac{-k^2 e(g_2 - g_1)(u_2 - u_1)}{[\log (Z_2/Z_1)]^2} \qquad (16)$$

where

E = evaporation in cm/s

e = density of air in gms/cm^3

g_2 and g_1 = specific humidities at heights Z_2 and Z_1 respectively

u_2 and u_1 = wind speeds (cm/s) at heights Z_2 and Z_1 respectively.

and K = Karman's constant $\doteq$ 0.41.

It should be again noted that this equation is strickly valid for neutral (steady-state) conditions. Under other conditions, such as periods of intense evaporation, use of the equation will give erroneous values due to the breaking of the logarithmic wind profile assumption.

Special equipment is required to make the necessary measurements of the variables in equation (16). Very accurate sensors have to be used to measure the small gradients of wind speed and vapour pressure over the short height interval. The temperature sensors must be able to read

correct to 0.05°C and humidity sensors to 0.01 mb. The anemometer must have minimum internal friction and a minimum time lag in response to wind fluctuations, they should (ideally) have a starting speed of less than 10 cm/s.

The Thornthwaite-Holtzman relationship is applicable to aerodynamically smooth surfaces such as lakes and reservoirs where the level of effective evaporation is taken as the water surface itself. On land, if the vegetation height is much different from the heights above the soil surface at which vapour pressure and wind speed measurements are made, the wind profile requirement can only be satisfied by applying a "zero displacement correction". This is due to the boundary from which turbulent transport is effective and is at some distance above the soil surface. An adoption (Rider, 1954) has been made to Thornthwaite's-Holtzman's relation to include a term for zero displacement. (Rider, 1954).

Empirical Formula:

Many attempts have been made to produce satisfactory formula for the estimation of evaporation. Usually they try to estimate evaporation from open water-surfaces but some others estimated potential and actual evaporation and transpiration from vegetated surfaces. Most of these empirical formulæ assume a constant supply of water hence estimates potential evapotranspiration.

The formula developed usually use data from meteorological measurements to compute evaporation or transpiration and are based on a combination of two or more of the estimation methods described above. Perhaps the most widely used approach for estimating evaporation from

meteorological factors is that based on a combination of the aerodynamic profile and the energy-budget methods. Of this type the Penman equation is the best known.

Penman Equation:

(Penman, 1948, 1952, 1956). Penman originally published his formula in 1948. It was used to estimate evaporation from free water surfaces from meteorlogical data. The formula requires the assumption of unlimited availability of water for evapotranspiration. Review of the technique in 1952 and 1956 resulted in lower and more accurate estimates for evaporation. Penman's approach is based on the measurement of the following four meteorological variables.

(1) Duration of bright sunshine

(2) Air temperature

(3) Air humidity

and

(4) Wind speed.

These four factors reflect most of the terms in the energy relationship (equation 15) but they do not take into account the energy reaching the body from the ground, rivers, precipitation and snowmelt for example nor do they show the effect of energy stored in the waterbody.

The simplified version of Penman's equation is given by (17)

$$E = \frac{\frac{\Delta}{\gamma} \cdot H + E_a}{(\frac{\Delta}{\gamma} + x)} \tag{17}$$

where

E = evaporation from surface. (E_o for open water and E_T for evapotranspiration).

Δ = slope of standard vapour and pressure curve at mean air temperature T_A.

γ = the constant of the wet and dry psychrometer equation

H = net radiation reaching the body

E_a = an expression for the drying power of the air

x = an expression indicating surface type. For open water, x = 1. For vegetated surface x > 1.

For open water, therefore, equation (17) becomes

$$E_o = \frac{(\frac{\Delta}{\gamma} H + E_a)}{(\frac{\Delta}{\gamma} + 1)} \tag{18}$$

Penman introduced a function which would vary with time of year and would enable direct estimation of evapotranspiration from estimated open water evaporation. This is given by equation (19) and f varies from 0.6 to 0.8

$$E_T = f \times E_o \tag{19}$$

It is not always possible to measure terms such as net radiation so Penman and others have set out empirical expressions for these cases.

E_a in equation (18) is an expression relating the wind speed at a height of two metres above the evaporating surface and the saturation deficit $(e_a - e_d)$ of the air at this height. Penman's formula for potential evapotranspiration is given by equation (20)

$$E_T = \frac{(\frac{\Delta}{\gamma} \cdot H_T + E_{AT})}{(\frac{\Delta}{\gamma} + 1)} \tag{20}$$

where

$$E_{AT} = 0.35\ (1 + \frac{u}{100})(e_a - e_d)$$

Thornthwaite Equation: (Thornthwaite, 1939, 1948, 1954)

The equation was developed in the United States using a statistical study of available observations. The dominant parameters in the appraoch are temperature and length of day. Thornthwaite intended that temperature would be a reasonable parameter to integrate the balance of radiation used for heating the soil and air and radiation used for evaporation. The Thornthwaite model is based on calculations of potential evaporation or transpiration, then, on the basis of a series of assumptions, the estimates of monthly runoff are made using established empirical rules.

Equation 21 lists the general Thornthwaite Equation for estimating potential evapotranspiration.

$$E = C.\ T_m^{\ a} \tag{21}$$

where

E = evaporation or potential evapotranspiration (water unlimited),

C = coefficient

T_m = monthly mean temperature (^oC), and

a = exponent

The constants a and c both depend on the location of the area. The exponent 'a' can be evaluated in terms of the annual heat index, I, as

$$a = 67.5 \times 10^{-8} I^3 - 77.1 \times 10^{-6} I^2 + 0.0179I + 0.492 \qquad (22)$$

in which

$$I = \sum_{m=1}^{12} \left[\frac{T_m}{S}\right]^{1.51} \qquad (23)$$

Thornthwaite simplified this relationship for the case of 12 hours of sunshine per day for a 30 day months. Equation (21) reduces to,

$$E = 1.62 \left[\frac{10\ T_m}{I}\right]^{a} \qquad (24)$$

For all other cases, tables are available which relate a correction factor for daylight hours to latitude. This correction factor is the constant 'c' in equation (21). Although the Thornthwaite equation is somewhat cumbersome to manually operate it does reduce to a simple nomograph [Gray, 1970] and lends itself easily for computer application.

<u>Blaney and Criddle Method</u>: [Blaney and Criddle, 1950]

This is similar to the Thornthwaite method in that it assumes that the heat budget is shared in fixed proportion between heating the air and evaporation. The formula was developed for the arid western United States and estimates comsumption use based on measurements of temperature and hours of sunshine. The monthly consumptive use, cu, is found by multiplying the mean monthly temperature, T_m, with the monthly percent of annual daytime hours, p, and a monthly crop coefficient, k.

(Equation (25))

$$cu = \frac{k \cdot T_m \cdot p}{100} = kF \qquad (25)$$

where $\frac{T_m p}{100}$ = monthly consumptive use factor

The seasonal consumptive use is obtained by summation of the relevant monthly values. The coefficient, k, is an average seasonal consumptive use coefficient selected for different crops.

The last three formula are based on the assumption that the rate of water use is unlimited, a condition seldom realised in nature. Several drawbacks exist for each type of method and these should be noted (Gray, 1970).

(1) The Thornthwaite and the Blaney and Criddle methods use the same data to arrive at an estimate of annual or seasonal consumptive use. Generally they both tend to give an overestimation of water use in the early growing season and an underestimation in mid-season, unless an appropriate crop factor is applied.

(2) The Blaney-Criddle method gives more reliable estimates of seasonal use than the Thornthwaite method for arid regions. They both are not as realiable as Penman's method for humid areas because of the absence of a humidity term.

(3) The Thomthwaite and Blaney-Criddle methods should not be used for estimations of water use for short periods of

time as no allowance is made in the methods for variations in wind and relative humidity.

(4) The Penman equation is hindered in its' application due to the number of meteorological observations required, however, it does provide the best means of estimating potential evapotranspiration. Some care should be used when applying to arid regions as advected energy may by significant.

Turc Equation: [Turc, 1954, 1955]

This formula was developed by assuming that water supply is not unlimited, but it is still basically a temperature formula. Turc carried out a comprehensive survey of general experience and found that usually evaporation is greater in wetter years than in drier. The first formula was designed to relate annual evaporation from catchment areas to precipitation and air temperature, the latter giving an indication of evaporation opportunity. The first Turc formula is given by equation (26)

$$E = \frac{P}{\left[0.9 + \left(\frac{P}{L}\right)^2\right]^{\frac{1}{2}}} \qquad (26)$$

where

E = evaporation (mm/annum)

P = precipitation (mm)

$L = 300 + 25T + 0.05T^3$

T = mean air temperature (^{o}C)

A more complex formula was latter developed for shorter time

periods were varying levels in soil moisture and crop factors are more pronounced. (Equation (27))

$$E = \frac{P + a + V}{[1 + (\frac{P+a}{\ell} + \frac{V}{20})^2]^{\frac{1}{2}}} \tag{27}$$

where

E = evaporation in a 10-day period (mm)

P = precipitation in the same period (mm)

a = estimated evaporation from bare soil during the same period assumming zero precipitation ($a \leq 10$)

V = a crop factor

ℓ = evaporation capacity of the air

$$= \frac{(T + 2)\, R_{si}^{\frac{1}{2}}}{16}$$

R_{si} = incoming radiant energy (cal/cm^2)

and T = mean air temperature over 10-day period (^oC)

The Cooking Lake Moraine has limited data on hydrological and meteorological variables. The results of this fact is the necessity of adopting procedures and techniques which are not the most accurate. Only two meteorological variables are available for any considerable length of time at a station close enough to be relevant to this study. These are the ninety-one years of record of maximum and minimum daily temperature and total daily precipitation recorded at Edmonton. This paucity of suitable records of meteorological variables limits the evaporation estimation technique to one which needs only temperature and precipitation. There are several such methods available but that devised by Thornthwaite has been selected.

CHAPTER 4

MONTE CARLO METHODS

Introduction

The Monte Carlo method can be defined as a technique which consists of generating a long series based on the characteristics of an available sample series. The method is also known as the experimental statistical technique. The definite process, in the Monte Carlo Method, is replaced by a random or stochastic process which will give the same result. It is worthy to observe that although a longer period of generated values are available, the information contained within the series is no more or no less than that of the sample series. Even with this limitation the technique has found various degrees of success in solving ~~the~~ numerous hydrologic problems [Sudler, 1927. Swanidze, 1964. Yevjevich, 1965]. It has of course had a longer and wider use in solving other probability problems such as, nuclear disintegration, cosmic clouds, telephone or traffic studies and in making random choices in some decision making process, say in playing a game. It is in fact probably correct to say that the experimental statistical method is as old as probability theory itself.

The first recorded example of the Monte Carlo process was probably that of Count Buffon's needle experiment of 1773. In this experiment Buffon observed that if a needle of length less than or equal to unity were tossed at random onto a horizontal surface ruled with equally spaced lines (at unit spacing) then the probability of the needle crossing the line is equal to $2L/\pi$, where L = spacing. Buffon therefore reasoned that he could determine the value of π experimentally by making

repeated trials. Another classic example of the Monte Carlo process, due this time to Roger Pinkham, is the experimental estimation of e. This experiment is based on the observation that if 2k numbers x_i, i = 1, 2k, are drawn in sequence from a random, equilikely source, then the probability that they are all in ascending order, that is, $x_1, x_2, x_3, \ldots x_{2k}$, is equal to 1/(2k)! If the probability of a failure, of an ascending sequence, on the odd number 2k + 1 is considered, that is, the difference

$$1/(2k)! \quad - \quad 1/(2k+1)!$$

it is possible to obtain the total probability that a sequency of drawings of random numbers from an equilikely source will produce a rising sequence that ends with an even number of numbers. This probability is,

$$\sum_{k=1}^{\infty} \left[\frac{1}{(2k)!} - \frac{1}{(2k+1)!}\right] = \sum_{k=0}^{\infty} \left[\frac{1}{(2k)!} - \frac{1}{(2k+1)!}\right] = \frac{1}{e} \tag{28}$$

From an experiment consisting of 252 runs a value of 1/e of 0.381 was obtained, this represents an error of 3.5% in the estimation of e.

Student in 1908 developed the idea of random sampling to estimate given distribution functions. Decks of cards and other sampling devices have been used to generate non-historic river flow patterns (Sudler, 1927) to achieve a mass diagram analysis to develope probability distributions of reservoir capacity. This was probably the first serious application of the Monte Carlo process to solve a hydrological problem.

Sources of Random Numbers

The first source of random numbers that likely springs to mind is some natural, physical phenomenon such as, cosmic rays, nuclear disintegration or perhaps sampling at random an alternating frequency. This technique of using a physical source has found many applications in past studies but has generally been superseded by computer methods which use some form of recurrence relation. A recurrence relation is where each successive number in a series is formed from the preceding number or in some relationships from several of the preceding numbers. Usually some form of a algorithm of arithmetic operations is applied. The most important advantages of the recurrent technique over the physical sampling are the repeatability of the experiment and the non-reliance on any physical properties particularly the stability.

The numbers produced from a scource number via a recurrence relation are predictable. In the true mathematical sense of random, the numbers simulated by the recurrence relation are not pure random numbers. To delineate then between the impossible, pure random numbers and the predictable random number, the term psuedo-random is used. The numbers therefore generated by a recurrence relationship are psuedo-random numbers. In practice it is of no consequence if the numbers are predictable, what is of importance is whether or not the use of the numbers is random.

Von Neuman has developed an algorithm method to generate psuedo-random numbers based on selecting the middle digits of products (Von Neuman, 1951). This particular generator is an example of a general

class of psuedo-random number generators which are most often used to-day and are commonly called multiplicative congruence generators. These are defined by equation (29)

$$X_{n+1} \equiv X_n \rho \ (\text{MOD } 2^k) \tag{29}$$

or as "X_{n+1} is congruent to X_n modulo 2^k" which means the difference between X_{n+1} and X_n is divisible by 2k.

Selection of ρ and Starting Value

First, it is obvious that if the starting value (X_o) is even, that is, divisible by 2, all the products will have at least one zero at the end. This effective wastes one bit of the machine capacity. Consequently X_o should be odd. If ρ is even, zeros will accumulate on the right hand side until

$$X_{k+1} = \rho^{k+1} X_o = 0 \ (\text{MOD } 2^k) \tag{30}$$

which results in all numbers past this point being zero. Therefore ρ is also an odd number.

Odd numbers can be written in one of the following four ways: (1) 8t + 3, (2) 8t + 1, (3) 8t - 1, and (4) 8t - 3, for some interger t. The value suitable for the integer t has been widely investigated (Hamming 1971, 1973) but would not be productive to reproduce the theory behind selection of t here.

All of the multiplicative congruent generators produce a series of uniformly distributed numbers. As this distribution is not one of the

more useful hydrological distribution the problem arises of how to obtain another distribution from a sample uniformly distributed. In principle the following device works. If the wanted cumulative distribution F(Y) of the distribution f(y) is equated to the cumulative distribution X of the flat or uniform distribution, solution of the equation will give numbers with the required distribution function. This is shown by the following equations.

$$\int_o^x 1 \cdot dx = X = \int_o^y f(y) \cdot dy = F(y) \tag{31}$$

Applying the inverse operator F^{-1}, gives

$$F^{-1} \{F(y)\} = y = F^{-1} (x) \tag{32}$$

As an example suppose the exponential distribution is wanted, that is, e^{-y}, then since,

$$f(y) = e^{-y}$$

$$F(y) = \int_o^y e^{-y} dy = 1 - e^{-y}$$

As, $F(y) = X$ by definition

$$e^{-y} = 1 - X$$

or $y = -\ln (1 - x)$

By using the sequence of psuedo-random numbers generated by a suitable generator, x_i, the series $y_i = -\ln(x_i)$ is obtained, with y_i having an exponential distribution. By inverting the cumulative function the normal distribution can be obtained except that the inverse function has

to be approximated, usually by auxiliary tables. An alternative method, used in this study, is to call on the central limit theorem. In this method several psuedo-random numbers are added from a uniform (or any other) distribution. Since each number supplied from the uniform distribution is independent of all other numbers generated and the variance of the series is,

$$\sigma^2 = \int_o^1 (x - \tfrac{1}{2})^2 \cdot dx$$

$$= \frac{(x - 1/2)^3}{3}\Bigg|_o^1 .$$

$$\sigma^2 = 1/12 \tag{33}$$

Therefore it can seem that the sum of twelve distributions will have a variance of one. Thus the common rule is to add twelve numbers from the generator and subtract six from the sum to get a normal distribution with mean of zero and a variance of unity.

A subroutine called grand is available in the University computer system, specifically the IBM Scientific Subroutine Package. The program has been writeen in assembly language but is essentually the same as subroutine RAND listed in appendix 2. This subroutine (grand) was used to generate a given length of precipitation and temperature data which follows a normal distribution with supplied mean and standard deviation. Grand computes twelve uniformly distributed numbers by the power residue method and with the central limit theorem obtains a psuedo-random number x conforming to the normal distribution. The random number x, will have the same mean and standard deviation as that supplied

to Grand. It was noted earlier that the starting value, x_o, should be odd if machine space is not to be wasted. In fact any odd number between 1 and 2**(31-1) can be used for the starting number in the University of Alberta machine. Any number can therefore be applied and facilities do exist to enable the programmer to add this but subroutine Grand supplies the starting value of 524287 if no other odd number is specified. This number was used for generating samples in this thesis.

CHAPTER 5

LAKE LEVEL ANALYSIS

As the title of this Chapter indicates it describes the analysis of lake level changes, obtained by the water-balance model, due to the variability of meteorological parameters in particular. The Chapter is made up of two parts. Part one deals with the proving of the model by estimating lake levels from the historical records of temperature and precipitation, and comparing these with lake levels measured. Part two extends the analysis by using data generated via the Monte Carlo Method (Chapter 4) and shows the possible large fluctuations that occur due to the natural random fluctuations in precipitation and temperature.

Analysis Using Historical Data

The method devised by Thornthwaite has been selected as the technique for estimating evaporation and evapotranspiration. Laycock has shown the applicability of the Thornthwaite technique for numerous water-balance studies within the Canadian Prairies. (Laycock, 1964, 1967, 1971, 1973). In his 1973 paper to the Symposium on the Lakes of Western Canada, Laycock reviewed the basic water-balance characteristics of the Cooking Lake Moraine and used Thornthwaites formula to estimate surplus and deficit amounts for Edmonton data.

Thornthwaite's equation is based on the mean monthly temperature. This gives an estimate of monthly potential evapotranspiration given an annual heat index composed of monthly heat indices. With known monthly

precipitation, storage capacity of the ground and antecedent storage condition estimates of monthly moisture surplus or deficit can be computed. Surplus conditions only occur whenever the difference between precipitation and evapotranspiration is positive. The surplus obtained can either runoff entirely or in part depending upon the antecedent storage condition. The runoff criteria can be defined by the following set of equations

$$S_n = S_{n-1} + P_n - E_n \tag{34}$$

$$R = \begin{cases} S_n - SC & S_n > SC \quad (35a) \\ 0 & S_n \leq SC \quad (35b) \end{cases}$$

where

S_n = storage condition of the soil horizons during month n, n=1, 12 (cms)

P_n = total monthly precipitation for month n (cms)

E_n = Thornthwaites estimate of monthly evapotranspiration (cms)

SC = storage capacity of the ground (cms)

R = runoff (cms)

If $S_n > SC$ there is runoff equal to the difference between the amount of water needed to fill the storage capacity and the amount of surplus. If runoff is greater than zero the storage condition of that month is set at SC. If $S_n < SC$ there has been some depletion in the storage condition of the ground, that is, evapotranspiration has exceeded

precipitation. As each month's runoff or depletion is based partly on the previous month's storage condition, the Thornthwaite procedure gives an on-going method which models dry and wet periods.

Master (appendix 2) is a computer program, written in Fortran, which produces estimates of monthly evapotranspiration given records of monthly mean temperature and total monthly precipitation and starting conditions. Various storage capacities can be assigned to the different geological, vegetal or topographical groups found in the Moraine. To accurately map the different storage conditions that exist throughout the Moraine would require a detailed investigation beyond the scope of this study. Consequently a typical or average value of four inches has been adopted to represent the unified storage capacity of the watershed. Laycock (Laycock, 1971, 1973) has shown that a storage capacity of four inches is representative of the majority of the watersheds in the Prairies and has applied this capacity to the Moraine area. A value of ten centimetres was adopted for this study to conform with S.I. units. A record of surplus and deficits for storage conditions (in steps of two centimetres) from 2 to 10 cms, and for 15 and 20 cms has been produced and are available if they should prove to be applicable later or when more input becomes available.

The analysis of the fluctuations of lake levels using historical data as input was first attempted by assumming a constant lake area to contributing basin area ratio. The results of this analysis showed that it was not correct to assume a constant ratio. By an inspection of the area-elevation curves (appendix 1) it can be seen that the value of the ratio of areas is not constant and can vary considerably. Any sudden

increases in lake levels could therefore not be accurately modeled if the area ratio were set at a constant value. This ratio, D, is defined as the ratio of lake surface area to the effective drainage area. Ratio D was changed by unit increments from a low of zero (no runoff at all) to 5, believed to be the maximum value. Figure 9 shows the lake levels of Cooking Lake for the six values of D and an arbitrary starting value of 100 m, also assummed to be the maximum lake level.

Interpretation of Figure 9 shows the fluctuations that have occurred due to water deficiency and surplus over the period of record. This rapid rise in lake level, reported during 1900 and 1901, is graphically shown. This sudden rise was due to the very heavy precipitation recorded during two consecutive years. The general "good condition" reported from the turn of the century to 1930 is shown by fairly stable lake levels. From 1930 to about 1970 a general decline in lake level can be seen for the higher values of D. It no runoff occurs, that is, D equals values of D zero, Cooking Lake would dry up completely in about forty years, assumming it starts from a full state several times the lake reached a maximum level for values of D greater than four. From this analysis the author expects that the actual ratio between lake surface area and effective drainage area is around three to four.

The adoption of a constant value for D is acceptable for a general appraisal of the lake level situation but it is not valid for the physical case. As the lake level increases and decreases over the months and years it is obvious to assume that the value of D will respectively increase or decrease. The next logical step is to try and model mathematically this change in D.

Area elevation curves are given in Appendix 1 for the major lakes.

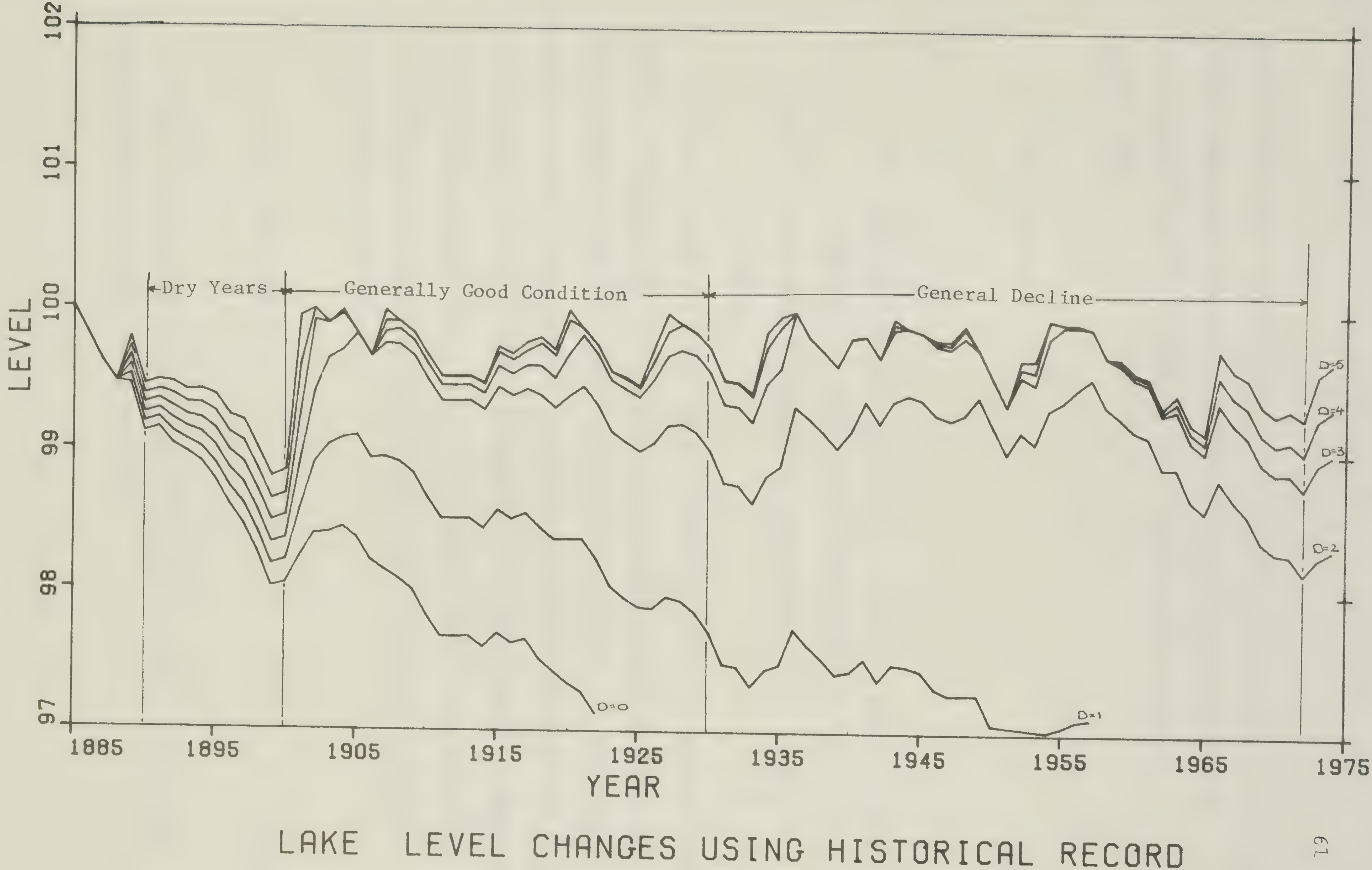


FIGURE 9: Lake Level at End of January for Cooking Lake

in the Cooking Lake Moraine area. In each case the area-elevation curve has been approximated by one or as two straight lines depending upon the individual case. The equation of a typical line is given by equation (36)

$$SA = A.C + B \qquad (36)$$

where

SA = surface area of the lake in acres for a given surface elevation, C, which is in metres above datum.

A and B = constants defining the linear relationship between SA and C.

The contributing area, CA, can be thought of as the effective drainage area and defined as that area of the total watershed which will contribute to runoff. This can easily be shown to be equal to the difference between the total drainage basin area(TDB) and the lake surface area (SA) multiplied by a factor (F) which depends upon the areal extent of depressions, marshes, phreatophytes etc, that is, those areas which do not contribute to runoff except possibly in very wet years. Mathematically the contributing area can be defined as,

$$CA = (TDB - SA) \times F \qquad (37)$$

Table 5 lists the drainage basin areas and the linear relationship between surface water area and lake level

TABLE 5

LINEAR RELATIONSHIPS BETWEEN LAKE SURFACE AREA (SA)

AND LAKE SURFACE ELEVATION (C)

LAKE	TOTAL DRAINAGE BASIN AREA TBD (ACRES)	RELATIONSHIP	RESTRAINT
Miquelon #1 and #2	14660.	SA=459.760*C-348607	C<762.8
		SA=270.848*C-204490	C≥762.8
Oliver	9600	SA=361.13*C-272168	C<756.1
		SA=270.85*C-203895	C≥756.1
Joseph	8400	SA-317.30*C-238981	All C
Ministik	22360	SA=1128.53*C-850912	All C
Cooking	46080	SA=3830*C-2807267	C<735.2
		SA=1459*C-1064344	C≥735.2
Hastings	26240	SA=361.13*C-263440	C<735.2
		SA=816.63*C-598305	C≥735.2

elevation. The restraint listed in column 4 of Table 5 is the elevation at which the area-elevation curve changes slope thus requiring a different straight line definition. The lines were all fitted by eye.

The determined contributing area will, for a given lake surface area, provide D times the surplus runoff estimated by Thornthwaite's procedure. That is, for R > 0

$$\Delta LL = D \times (R) - PE \tag{38a}$$

$$= \frac{(TDB - SA)}{SA} \times F \times R - PE \tag{38b}$$

and

$$\Delta LL = -E \quad \text{for } R \leq 0 \tag{39}$$

where

R = run off

ΔLL = change in lake level when surplus runoff occurs

PE = estimate of potential evaporation

and all other terms as defined earlier.

It is worthly to note that assumed maximum and minimum lake levels have already been given in Table 4. For completeness, the relative areas of forests and total watershed area for each drainage basin is given in Table 6.

For each major lake in the system a computer run (using MASTER) was made and levels for each month of the year estimated. The levels in October for each lake have been plotted with help from the Computing Services subroutine, CGPL. The plot of estimated lake levels using historical data for each lake are similar in shape but the fluctuations can be larger or smaller depending upon the lake in question.

TABLE 6

DRAINAGE BASIN CHARACTERISTICS OF THE COOKING LAKE MORAINE

DRAINAGE BASIN	TOTAL DRAINAGE BASIN AREA	TOTAL WATER* AREA (AC)	FORESTED AREA (AC)	CLEARED LAND AREA (AC)
Miquelon Lakes #1 and #2	14,600	2,710	7,620	4,330
Joseph Lake	8,400	1,720	1,500	5,180
Oliver Lake	9,600	910	8,090	600
Ministik	22,360	2,790	14,030	5,540
Cooking Lake	46,080	10,470	18,150	17,460
Hastings Lake	26,240	2,880	12,460	10,900
TOTALS	127,340	21,480	61,850	44,010

* Areas of forested cleared lands and water areas taken from 1974 aerial mosaics prepared by the Land Use Sub-Committee.

Actual records of lake levels are available for Cooking Lake and the plots of lake levels for this lake have been analysised. The plots of the other lakes are given in appendix 4, and for Cooking Lake in Figure 10. The other variable in the determination of lake level changes is the factor, F, which measures the extent of contributing area relative to total area. The factor takes into account the losses to runoff due to depressional storage, snow drifting and phreatophytic use. Again Master was used to estimate lake levels with F = 0.25 up to F = 1.0 incrementing in steps of 0.25. Figure 10 shows estimated lake levels obtained with F = 0.65. Figure 11 shows the October lake levels estimated by using the four values of factor 'F'. It can be seen that the recorded lake levels seem to fall between the estimated lake levels when F = 0.5 and F = 0.75, hence the selection of 0.65 for 'F' when producing the levels shown in Figure 10.

Records of actual lake levels from 1956 to the present time are available for Cooking Lake. As a value for the lake level in 1885 is unavailable, it is necessary to assume the level and to relate the estimate with the measurements for thet particular starting value. After several trials a starting lake level of 736.5 m above ordinance datum was found to give the closest approximation when comparisons were made.

The fluctuations, illustrated by Figure 10 can occur sharply, particularly sudden rises in level. These will occur whenever an exceptionally wet year or a series of above average rainy years are encountered. The estimated values of the Cooking Lake level follow the measured fluctuations quite reasonably, but do debiate on a few occasions considerably from the actual lake levels. More records and surveys would both improve the estimates and give a better test of the model. The devia-

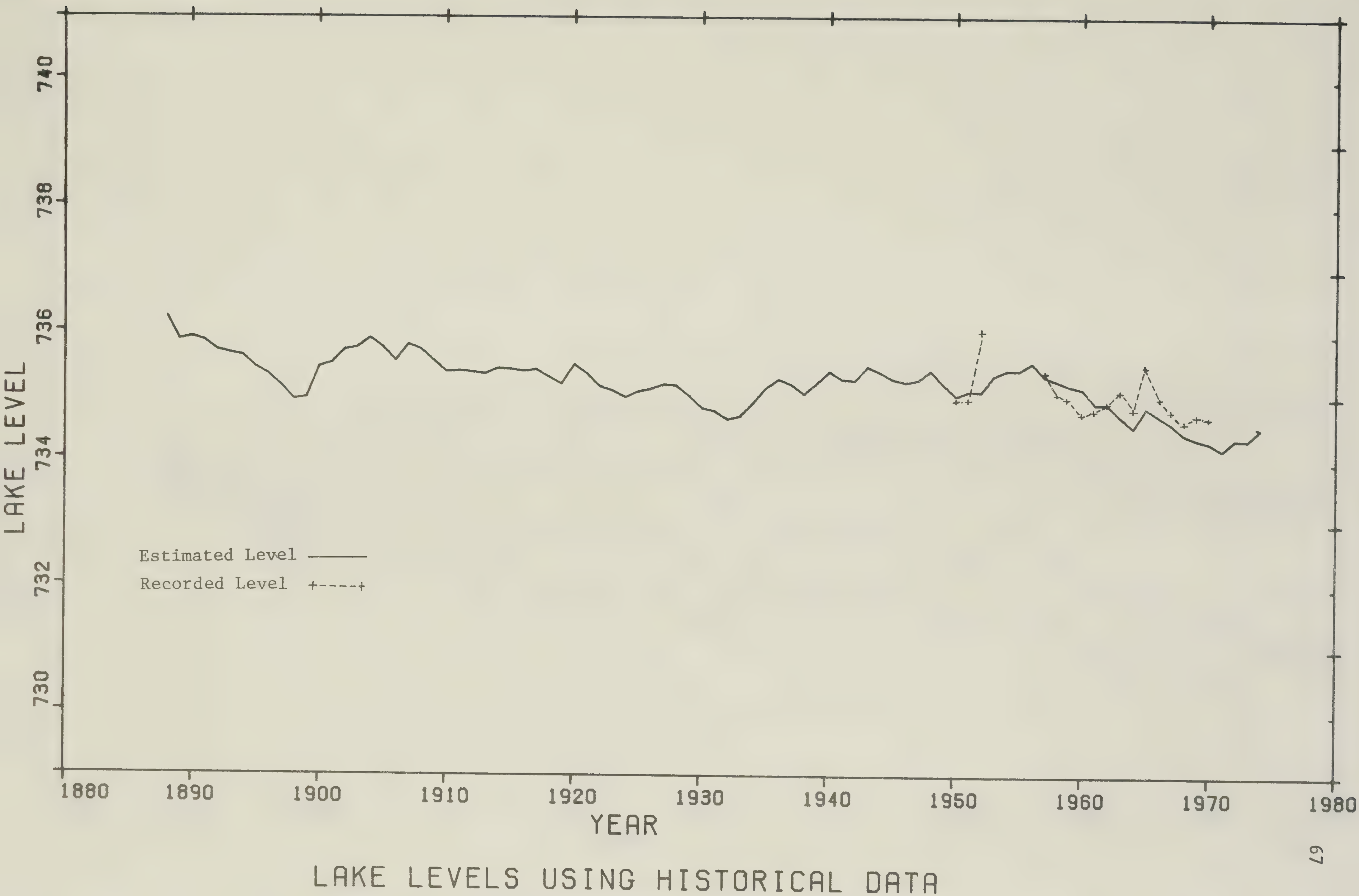


FIGURE 10: Cooking Lake October Lake Levels (F = 0.65)

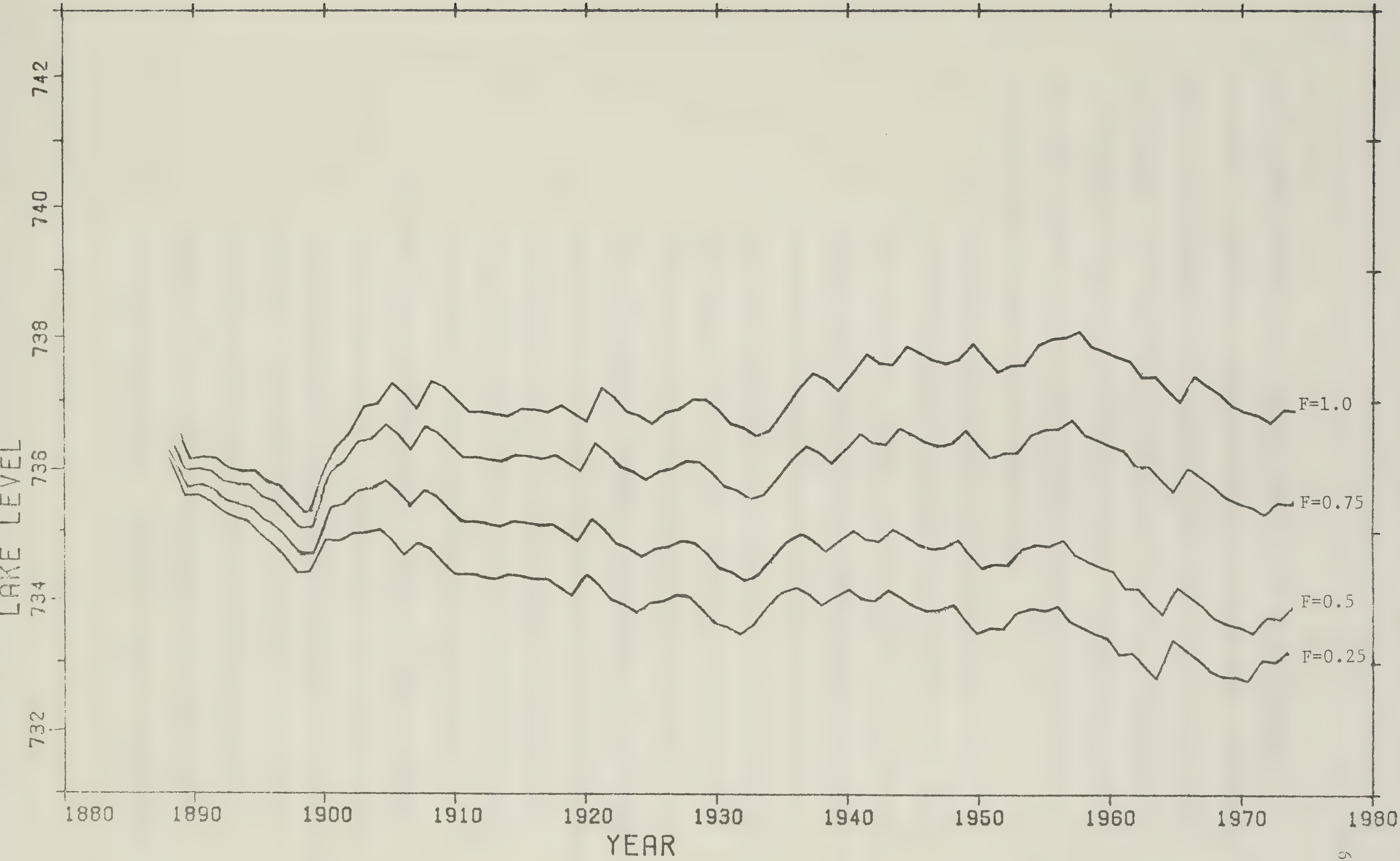


FIGURE 11: Cooking Lake Levels for October (Variable F)

tions between estimated and measured lake levels can be partially explained if the physical characteristics of the basin are investigated. Such an investigation will undoubtedly reveal numerous cause and effect relationships and at least several hypotheses could be postulated which could adequately explain the difference in model predicted lake levels and those recorded. Basic to most of the hypotheses that could be postulated would be the following observations.

(1) The nature of the model is such that errors in the evapotranspiration estimation (hence runoff) for a given time period are ignored. This is due to the unavailability of the essential hydrological and meteorological information necessary to produce a model which can estimate runoff and model the physical properties of the basin correctly. The assumption of an average capacity of 10 cms may not be valid for most of the sub-basins in the Moraine. It is also generally accepted that Thornthwaite's procedure gives an over estimate of evapotranspiration therefore under estimating runoff. It would therefore be expected that higher lake levels could be predicted if a better method of estimating evapotranspiration becomes available.

(2) The effective drainage area factor, F, cannot be thought of as a constant, for it obviously varies from one year to the next depending upon the relative amounts of precipitation and evapotranspiration that have occurred. This factor would also be expected to have a yearly variation with a maximum in the spring and a minimum in the winter when no runoff occurs at all. Within the summer rainfall period

this factor is also believed to have a significant variation. This summer variation being dictated by the duration, intensity and spatial arrangement of precipitation within the basin.

(3) The dependence of runoff on the amount and more significantly, with respect to lake level changes, the location of rainfall within the watershed will also have its effect on the magnitude of lake level fluctuations. The spatial variation of rainfall and the topographic features of the basin such as areas of bogs, local depressions, drainage channels and outcrops will combine to further help in either the over or under-estimation of runoff amounts.

(4) The amount of snowfall within the basin is a random variable which has been modeled by showing, with a suitable transformation, that it follows the normal distribution. Snow has however the unfortunate property of drifting into coulees and depressions whenever a wind blows. These deep banks localize runoff so that they yield concentrated pockets of snowmelt runoff. The extent of drifting will obviously vary from year to year depending upon the wind pattern and time of snowfall. This introduces another degree of unpredictability into the model and can only be assimulated into the water-balance model by physical measurement either directly by snow surveys or indirectly by one of the photogrammetric techniques.

(5) Contributions, from other lakes, to the water-balance of a lake are controlled by estimates of maximum lake levels. These estimates were made from topographical maps with a large contour interval. Consequently the contributions from one lake to another will not be correctly represented. It is believed that the method used underestimates the contribution from overflow and so, particularly for primary lakes lower down in the system, higher lake levels should be expected.

This discussion could obviously be extended to include many of the other hydrological and meteorological variables affecting the hydrology of the Cooking Lake Moraine. Such further elaboration on these topics is not thought to be necessary as it is accepted that the water-balance model developed here has limited suitability for accurate lake level estimates due to the limited data available. However, the fluctuations in lakes levels, in particular the changes due to the randomness of natural meteorological variables such as temperature and precipitation, can be illustrated using the model and data generated via the Monte Carlo Method.

Simulated Lake Levels

Many physical phenomena have a random or stochastic variability with a deterministic (often dominant) component which can be exactly defined. In nature many of the deterministic components of physical phenomena, such as lake levels, are influenced by man's everday life and the style of such a life. In the Cooking Lake Moraine man has altered the physical properties of the watershed by clearing land, developing agricultural lands and recreation areas, and diverting water to places outside of the natural boundaries. Developments of this kind can obviously have compensating effects when viewed comparatively but they do undoubtedly change the deterministic component of the many variables effecting the hydrology of the area.

The alteration of the measurable component of a random variable can be subtle, such as increasing use of fertilisers could increase nutrient levels in surface and sub-surface runoff, and so enhance eutrophication consequently changing the evaporation regime. Or, the alteration can be drastic and possibly irreversible such as gross water diversions or drainage of marshy lands. Such a drastic change in the deterministic component of the lake level of Miquelon Lake occurred in 1927 when Camrose built a canal, partially emptying the lake.

It is not possible to know what future flows in rivers or what precipitation events will be, but it is probable that future events will have the same stochastic properties as the observed historical record. This assumption forms the basis of stochastic hydrology and it is from this premise that lake level fluctuations are simulated. This model is

not intended to predict lake levels of the future because the effects and occurrence of the many variables are random and so are unpredictable, but the model shows that large variations in lake levels can be explained from meteorological variability.

Program, MASTER, is adaptable to producing a long record of potential evaporation, surplus and deficit amounts, and lake level estimates by including a call to GRAND (page 55). When the values of the mean and standard deviation of the transformed variable, total monthly precipitation and the mean and standard deviation of mean monthly temperature are fed into the model a series of monthly lake levels for a specified period (500 years) is produced. Due to a limitation on the plotting subroutine only 500 points can be plotted. This limits the plotted output to either, approximately forty years of month to month fluctuations or to a plot of the lake level in a particular month for 500 years. The latter format has been adopted.

Using the Monte Carlo Method described in Chapter 4 a 500 year series of total monthly precipitation and mean monthly temperature was obtained. This generated series is then used as the input data for the water-balance model so that, estimates of monthly lake levels can be plotted for a given month. An inspection of the simulated lake levels obtained from this input of randomly generated meteorological data can now be made. The simulated lake levels for October are shown in Figure 12 for Cooking Lake and Appendix 4 for the other major lakes.

The simulated lake levels correspond with the historically estimated lake levels in several general features. The first similarity is the occurrence of sudden increases in lake levels spanning a relatively short period of several years. These sudden fluctuations are similar to the

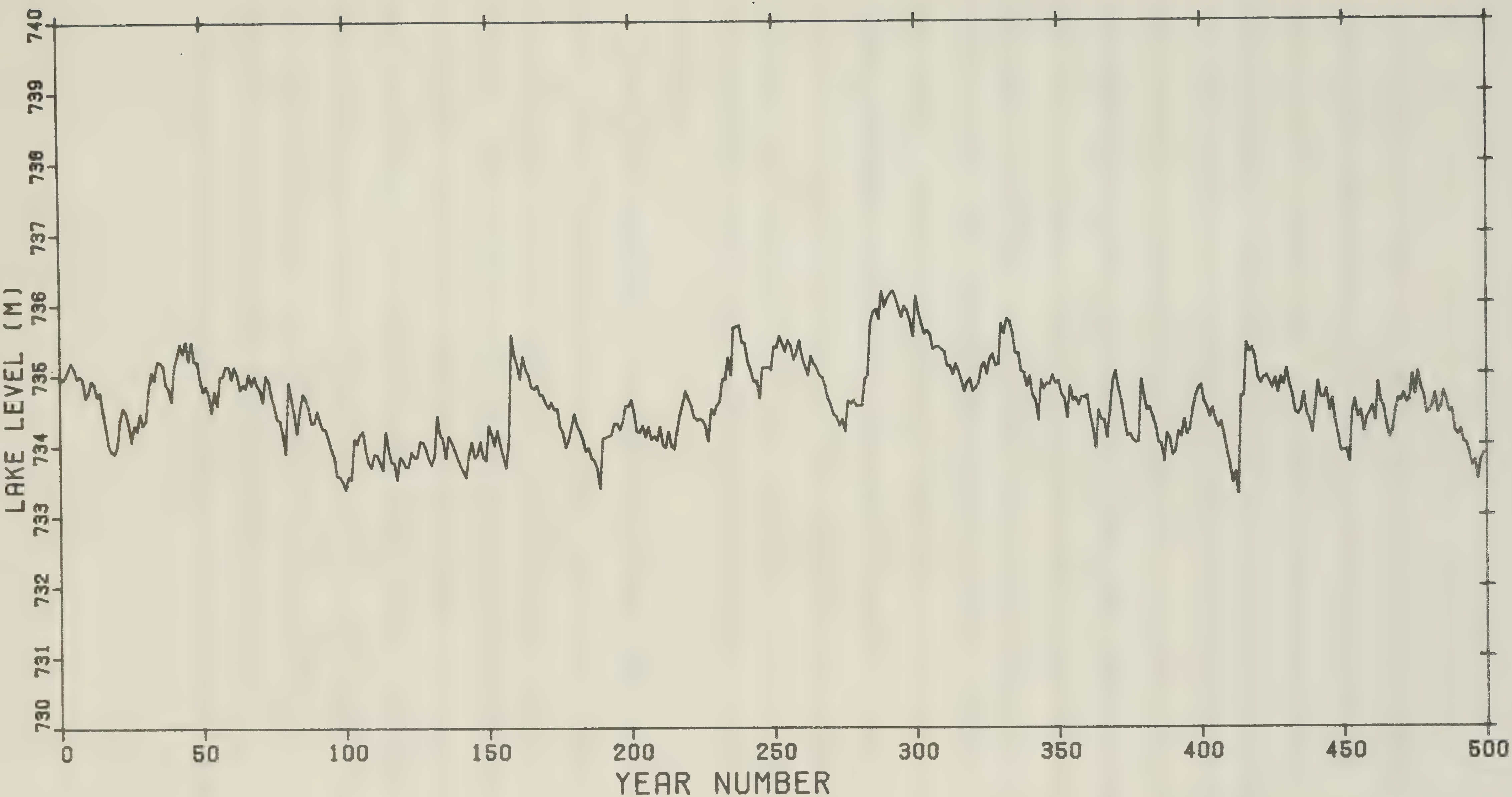


FIGURE 12: Simulated October Lake Levels for Cooking Lake (F = 0.65)

observed rise in the lake level recorded at the turn of this century. However, these rises are not of the same magnitude nor of the same magnitude as the observed rise. Over the five hundred year period simulated, three of these sudden restorations in lake levels occurred. The first occurred about year number 165 (Figure 12) and this rise in level culminated a relatively stable lake level period which had last approximately fifty years. The second and third rises however, occurred after the lake levels had been in a declining phase for approximately forty years in the first case and eighty years in the second. The second and third sudden rises occurred approximately at year 280 and 410 respectively. Positive fluctuations in level did occur during the declining phases mentioned above but they did not effectively cause a reversal in the declining trend. It would appear from the inferences made above that a declining trend can only be reversed or halted by the occurrence of an exceptional hydrological event such as a very wet year or a series of wet years.

After each restoration of the lake level by heavy runoff the system seems to decline towards an equilibrium level. This can be observed for both the historical plot of lake levels figure 10 and the simulated plot, figure 12 . When the lake system reaches its apparent equilibrium level it fluctuates randomly for a varying time period until another exceptional rainy period occurs. The system does not appear to have a definite trend towards either a high lake level or to a low lake level. Large fluctuations, both positive and negative, can be explained by the occurrence of exceptional hydrological events such as droughts and floods.

The decline of the lake system contained within the boundaries of

the Cooking Lake Moraine, over the last seventy years, can be explained, not from an investigation of land-use changes but from a more basic explanation based on the variability of natural phenomena. The simulated lake levels provide some evidence for a belief that the lake system prefers to decline from a high lake level to an equilibrium level around which it will oscillate until interrupted by an extreme wet year or wet years. This extreme hydrological event restores the lake system to an unstable high level from which the cycle starts again. However, this cycle cannot be assigned a certain periodicity as the culmination of a declining stage or equilibrium stage is based solely on the occurrence of a random event.

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The water-balance model developed for the Cooking Lake Moraine, shows the response of a lake system (with assumed steady-state physical properties) when excited by mathematically generated random meteorological variables. The response of such a system to a random input has been shown to be governed primarily by the variability of the natural random variables of precipitation and temperature. As a consequence of this dependence of the lake levels on the natural variability of climate, the response of the system is also random.

An investigation was made to determine the frequency distribution of the random variables, total monthly precipitation and mean monthly temperature. It was shown that after determined transformations, the distribution of the two random variables can be fitted to the normal distribution.

The characteristics of the simulated lake levels are similar in general features to a series of lake levels obtained by using historical data as input to a water-balance model. Lake levels appear to follow a cycle starting with a high unstable level. This unstable state is followed by a declining phase which eventually leads to an oscillatory stage where increases in level roughly balance the decreases. The latter two stages are subject to interruption, at any time, by an exceptional hydrological event. This event restores the lake level to a high unstable state and so effectively restarting the cycle. However no definable period can be assigned to this cycle as the exceptional hydrological event can occur at any time.

The effects of land use changes are not thought to play an important role in lake level fluctuations. However, land use changes affect the physical properties of the watershed and so the deterministic component of the water-balance model. Land use changes may affect the long term condition of the lake system by altering the evapotranspiration regime of the watershed. The water-balance components of precipitation, evapotranspiration and runoff have the more significant effect on lake level fluctuations. The natural variability of precipitation and temperature impose on the lake level fluctuations a character which is more changeable or random than would be expected solely from land use alterations.

Recommendations

It is necessary in any future study to develop more refined methods in measuring the physical characteristics of the watershed. The properties of the basin which should be better defined include, (a) the areal extent of differing vegetal groups and soil types, (b) areas of depressional storage which do not contribute to lake runoff, and (c) the accurate survey of outlet control levels of each lake. If this elaboration was achieved, the water-balance model would be greatly improved with respect to both accuracy and reliability.

A better evaporation estimation procedure is available in the Penman Method but it is believed that if a better model of the physical character of the watershed could be developed, the Thornthwaite procedure would still yield valuable results.

The Monte Carlo Method has been used to generate normally distributed psuedo-random variables with a given mean and standard deviation. As it is possible that significant correlation could exist between meteorological

variables measured in consecutive months it would be advantageous to investigate this possibility in any future studies. If some correlation did exist to some considerable level the Monte Carlo Method should contain the lag-1 serial correlation coefficient.

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APPENDICES

APPENDIX 1

AREA - CAPACITY CURVES

Joseph Lake

Hastings Lake

Miquelon Lake No. 1

Ministik Lake

Oliver Lake

Cooking Lake

JOSEPH LAKE

AREA-CAPACITY CURVE

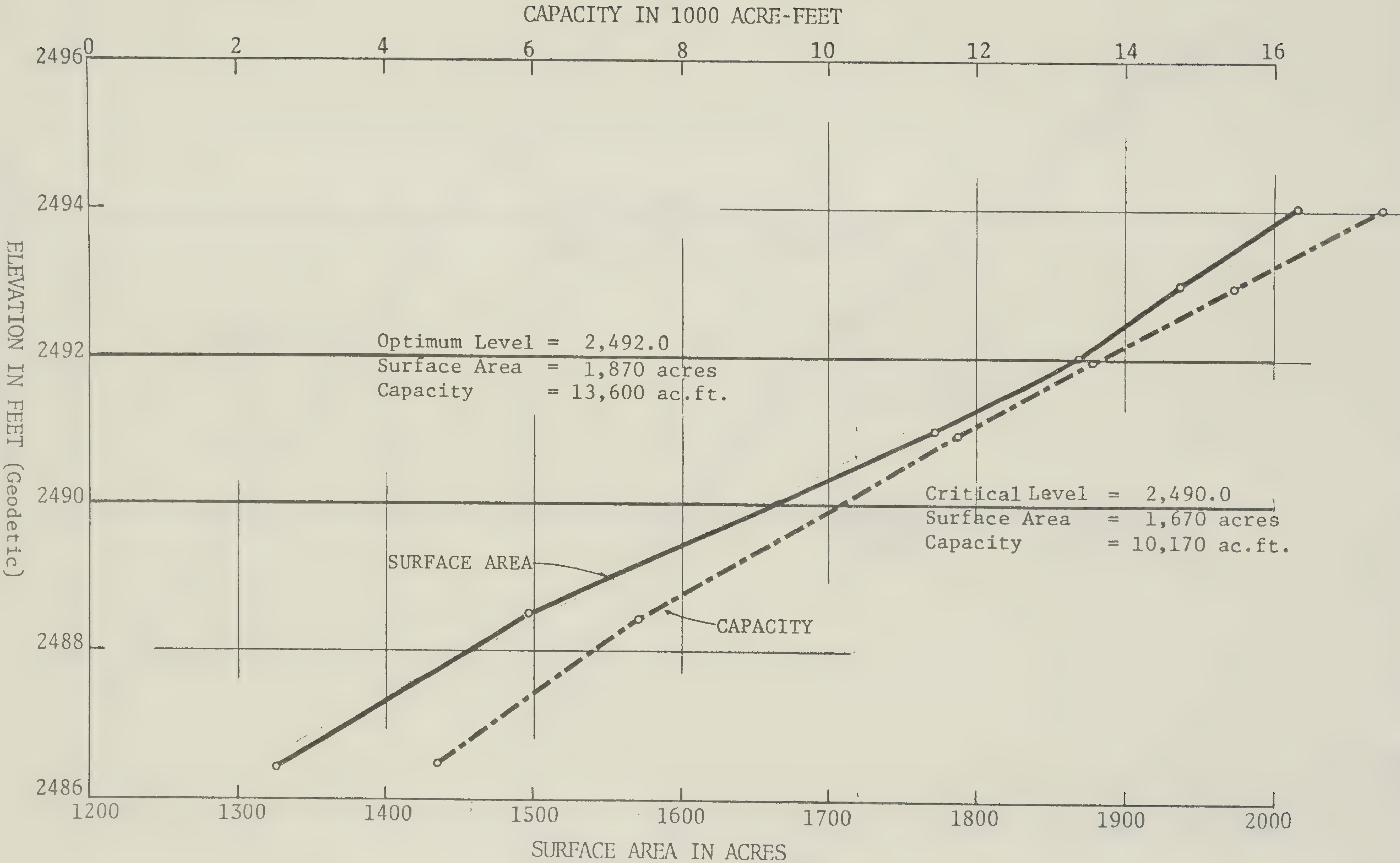


FIG. A

HASTINGS LAKE AREA-CAPACITY CURVE

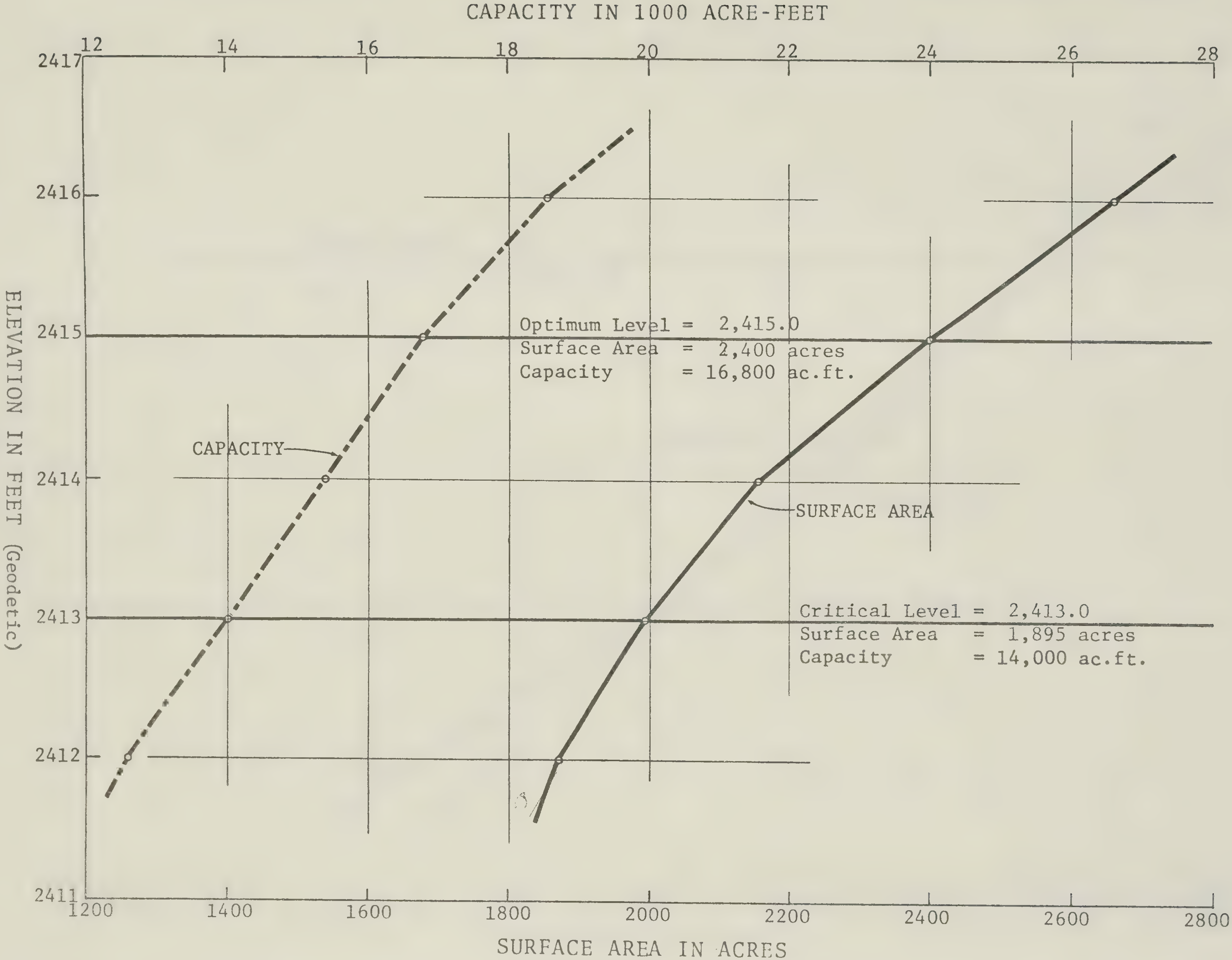
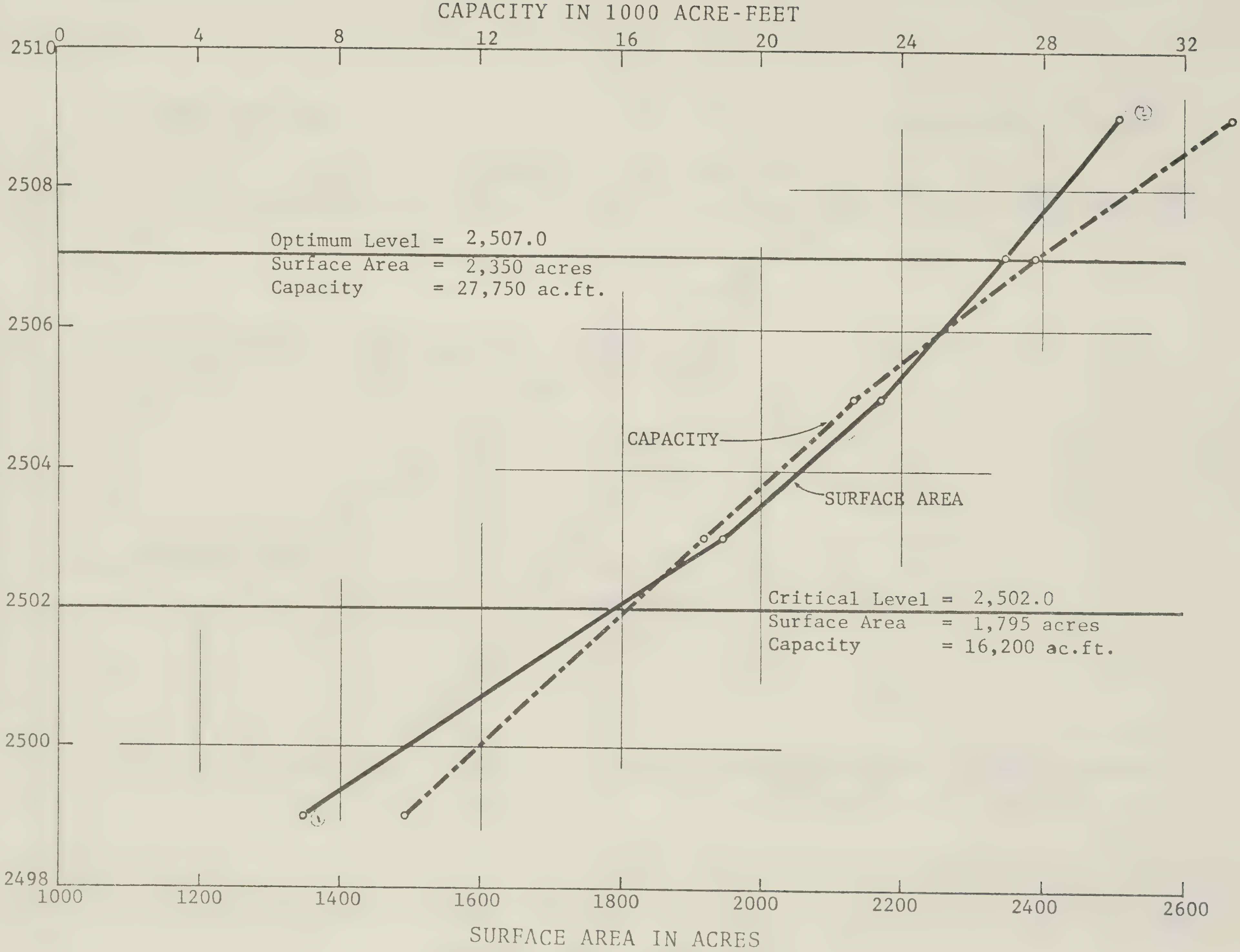


FIG. B

MIQUELON LAKE NO. 1

AREA-CAPACITY CURVE



ELEVATION IN FEET (Geodetic)

FIG. C

MINISTIK LAKE

AREA-CAPACITY CURVE

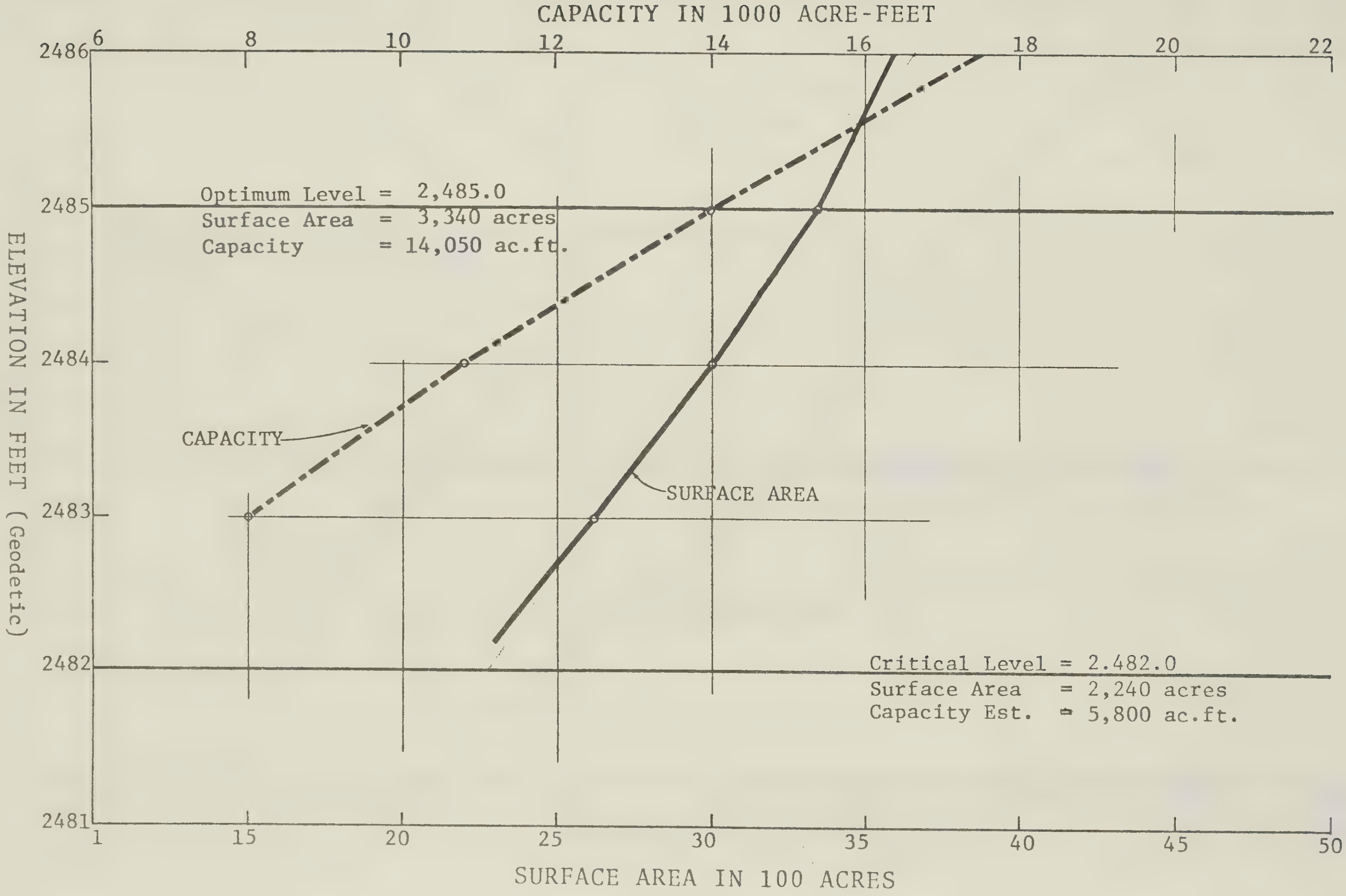


FIG. D

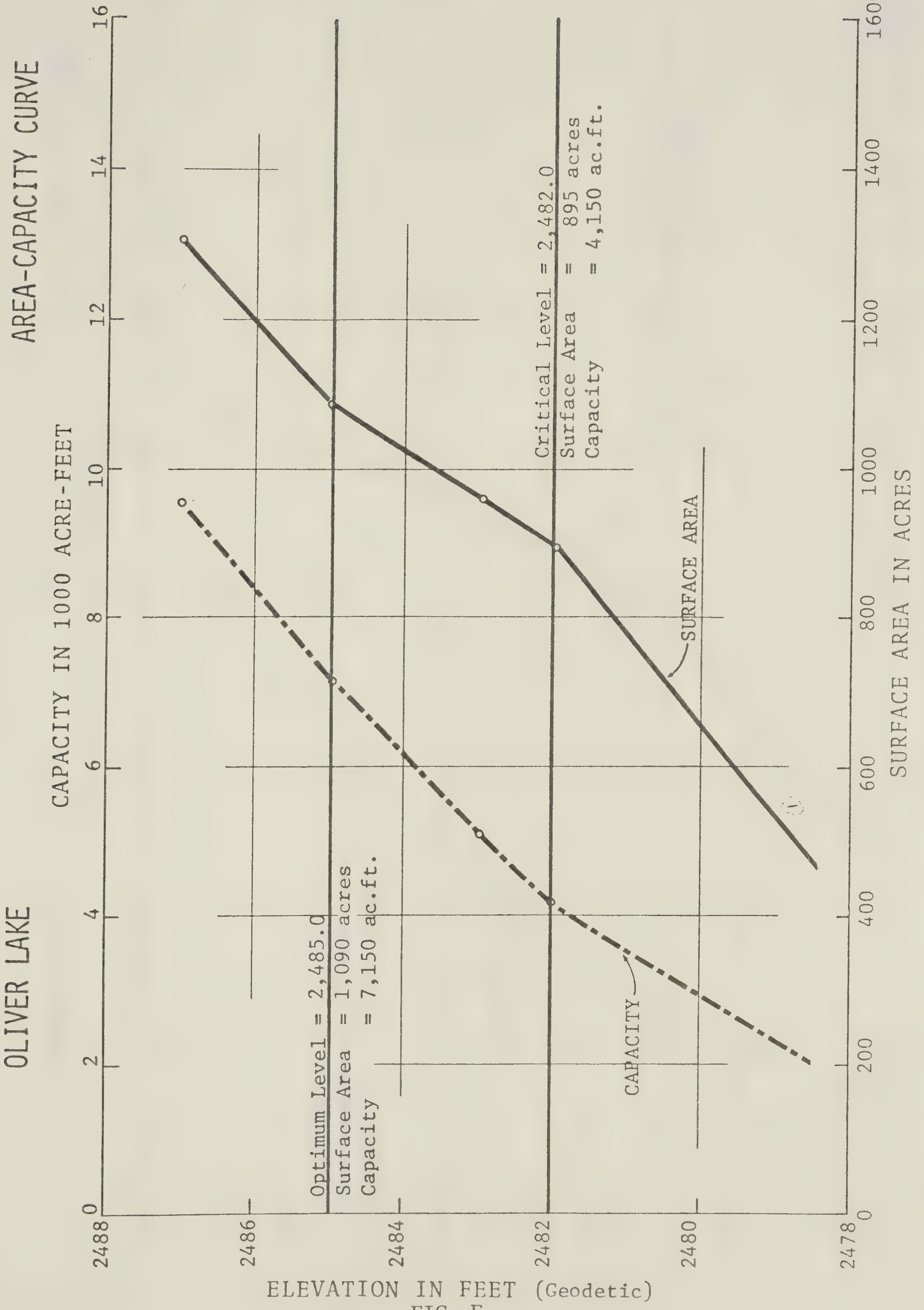

OLIVER LAKE
AREA-CAPACITY CURVE
CAPACITY IN 1000 ACRE-FEET
SURFACE AREA IN ACRES
ELEVATION IN FEET (Geodetic)
Optimum Level = 2,485.0
Surface Area = 1,090 acres
Capacity = 7,150 ac.ft.
Critical Level = 2,482.0
Surface Area = 895 acres
Capacity = 4,150 ac.ft.
SURFACE AREA
CAPACITY
0
2
4
6
8
10
12
14
16
200
400
600
800
1000
1200
1400
1600
2488
2486
2484
2482
2480
2478

FIG. E

COOKING LAKE

AREA-CAPACITY CURVE

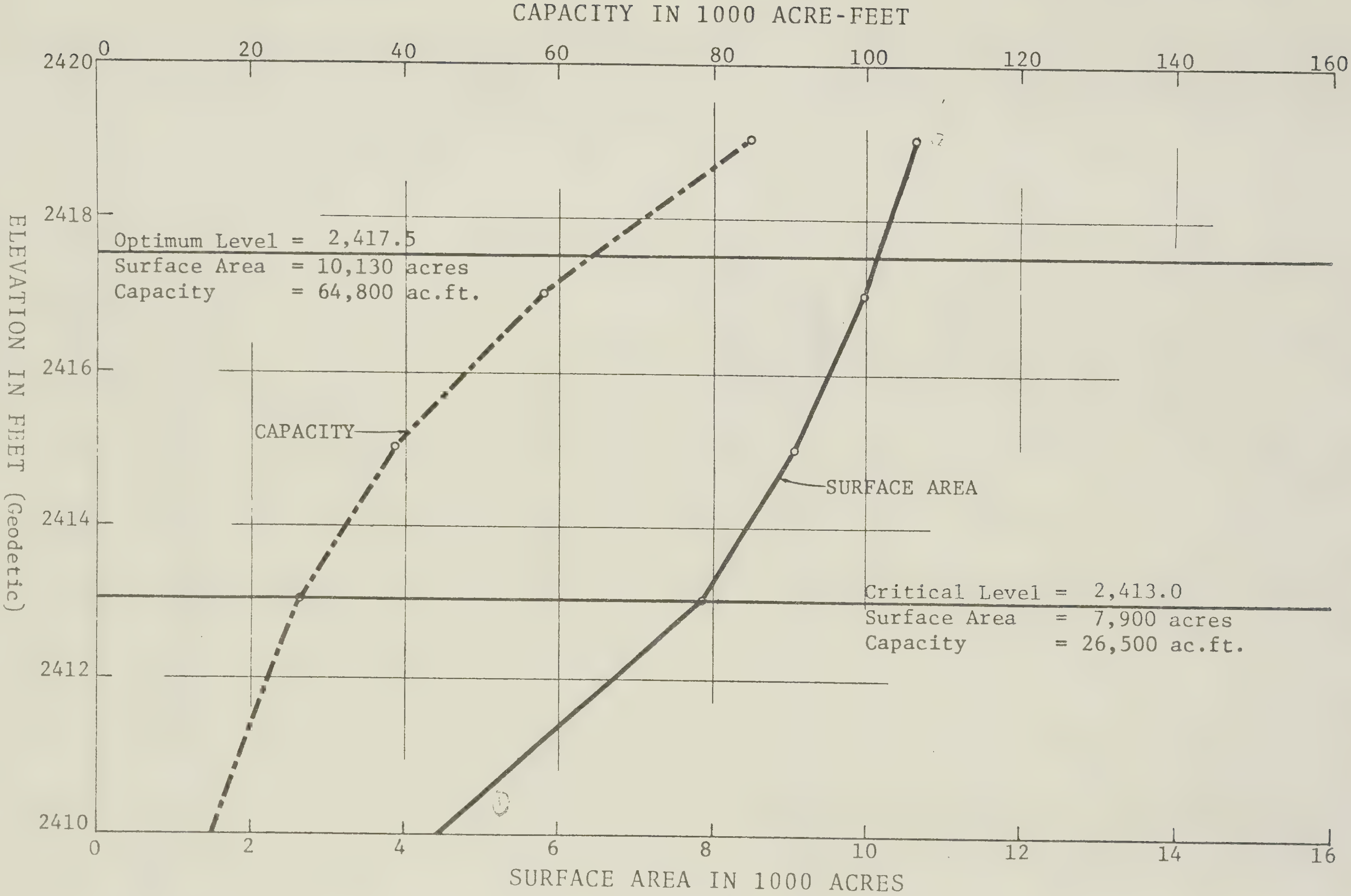


FIG. F

APPENDIX 2

COMPUTER PROGRAMS

Main Computer Program - MASTER

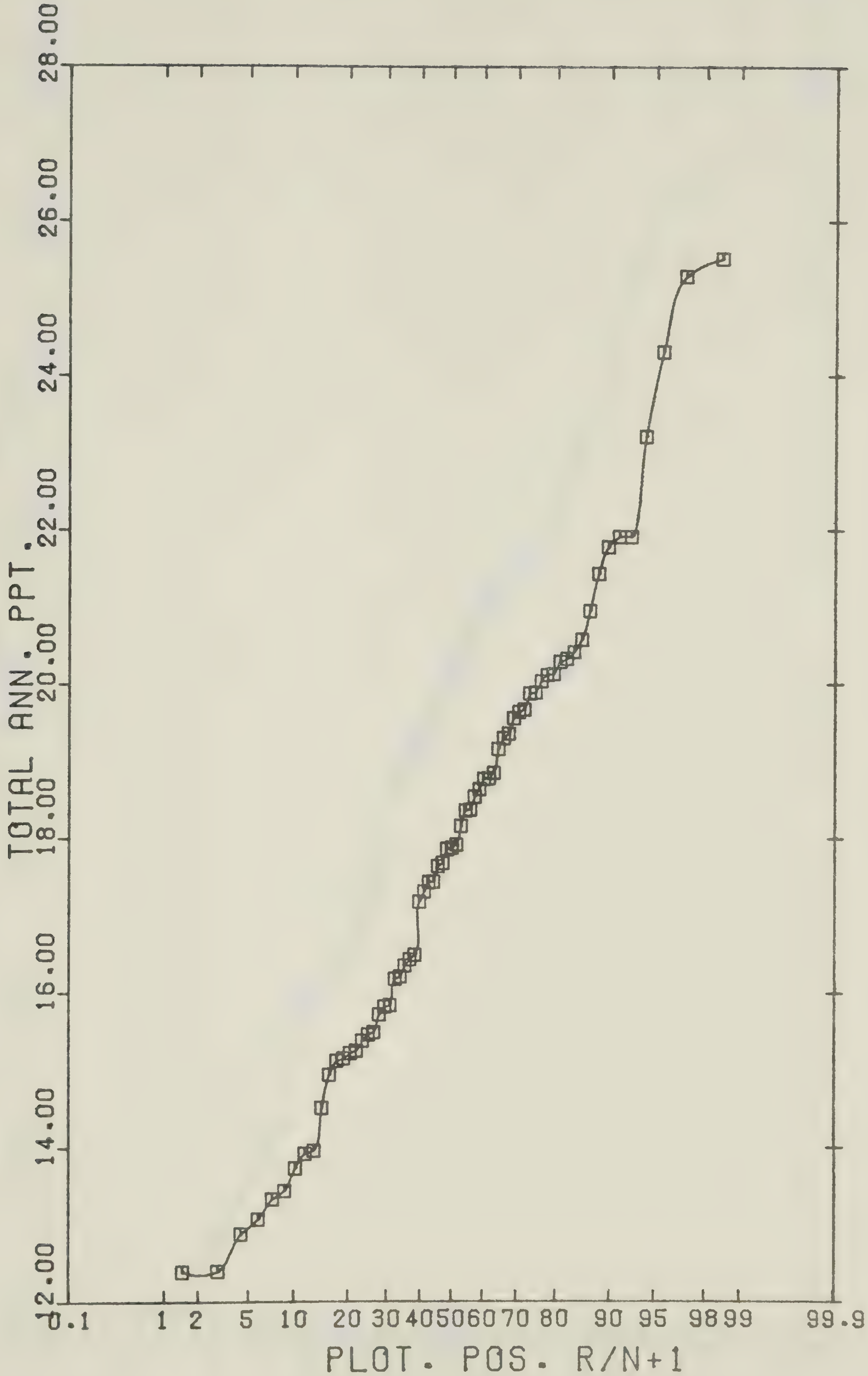
```
      DIMENSION PPTBAR(12),STPPT(12),PPT(12,500),TEMP(12,500),
     *AHI(500),TEST(12),HRS(12),HI(12),A(500),E(12,500),PE(12,500),
     *SURP(12,500),SC(12,500),DEF(12,500),STOR(13,500),BET(12),
     *SCHIL(500),CHLL(12,500),AL(12,500),B(500),X(501),ALPHA(20),
     *ALMAX(6),ALMIN(6),C(6),REST(6),TECA(6),ALOW(6),TEBAR(12),
     *AUP(6),T(500),P(500),BLOW(6),BHI(6)
C READ NUMBER OF YEARS GENERATED, AND THE VALUES OF THE MEAN
C AND STANDARD DEVIATION OF TOTAL MONTHLY TRANSFORMED PPT.
      READ(5,499)N,MON
  499 FORMAT(2I5)
      READ(5,505) (ALPHA(I),I=1,20)
  505 FORMAT(20A4)
      READ(5,500) (PPTBAR(J),J=1,12)
      READ(5,500) (STPPT(J),J=1,12)
  500 FORMAT(12F8.4)
C GENERATE N YRS OF NORMALLY DISTRIBUTED VALUES OF TRANSFORMED PPT.
      IX=65539
      DO 100 I=1,12
      DO 101 J=1,N
      CALL GAUSS(IX,STPPT(I),PPTBAR(I),P(J))
  101 PPT(I,J)=P(J)
  100 CONTINUE
C READ MEAN AND STANDARD DEVIATION OF MEAN MONTHLY TEMPERATURE
      READ(5,500) (TEBAR(I),I=1,12)
      READ(5,500) (TEST(I),I=1,12)
C GENERATE N YRS OF MEAN MONTHLY TEMPERATURE
      IX=524287
      DO 102 I=1,12
      DO 103 J=1,N
      CALL GAUSS(IX,TEST(I),TEBAR(I),T(J))
  103 TEMP(I,J)=T(J)
  102 CONTINUE
C READ IN NUMBER OF POSSIBLE HOURS OF SUNSHINE FOR EACH MONTH AT 5(
      READ(5,504)HRS
  504 FORMAT(12F6.2)
      DO 104 J=1,N
      AHI(J)=0.0
C DETERMINE ANNUAL HEAT INDEX
      DO 105 I=1,12
      TEMP(I,J)=(TEMP(I,J)-32.)*5./9.
      IF(TEMP(I,J).LE.0.0)GOTO10
      HI(I)=(TEMP(I,J)/5.)**1.51
      GOTO105
   10 HI(I)=0.0
  105 AHI(J)=AHI(J)+HI(I)
      CTO=0.0179*AHI(J)+0.492
      A(J)=(67.5E-08)*AHI(J)**3-(77.1E-06)*AHI(J)**2+CTO
C ESTIMATE MONTHLY POTENTIAL EVAPORATION BY THORNTHWAITE'S METHOD
      DO 106 I=1,12
      IF(TEMP(I,J).LE.0.0)GOTO11
      E(I,J)=1.6*(10.0*TEMP(I,J)/AHI(J))**A(J)
      PE(I,J)=HRS(I)*E(I,J)
      GOTO106
   11 PE(I,J)=0.0
  106 CONTINUE
```

```
  104 CONTINUE
C WRITE MONTHLY ESTIMATES OF PE
C       WRITE(6,600)
C 600 FORMAT(1X,'POTENTENTIAL EVAPOTRANSPIRATION'/3X)
C       WRITE(6,601)PE
C 601 FORMAT(1X,12F8.2)
        READ(5,500)BET
C READ IN MONTHLY TOTAL PPT. TRANSFORMATIONS
C TRANSFORM GENERATED PPT. VALUES TO NATURAL VALUES
        DO 107 J=1,N
        DO 108 I=1,12
        IF(PPT(I,J).LE.0.0)GOTO12
        BET(I)=1.0/BET(I)
        PPT(I,J)=PPT(I,J)**BET(I)*2.54
        GOTO108
     12 PPT(I,J)=0.0
    108 CONTINUE
    107 CONTINUE
C ESTIMATE CHANGE IN STORAGE
        DO 109 J=1,N
        DO 110 I=1,12
    110 SC(I,J)=PPT(I,J)-PE(I,J)
    109 CONTINUE
C SET STORAGE CAPACITY OF GROUND AND STORAGE CONDITION AT START
        COVER=10.0
        AST=10.0
C ESTIMAT SURPLUS OR DEFICIT FOR EACH MONTH
        DO 111 J=1,N
        STOR(1,J)=COVER
        DO 112 I=1,12
        STOR(I+1,J)=STOR(I,J)+SC(I,J)
        IF(STOR(I+1,J))5,6,7
      5 DEF(I,J)=-1.0*STOR(I+1,J)
        SURP(I,J)=0.0
        STOR(I+1,J)=0.0
        GOTO112
      6 SURP(I,J)=0.0
        DEF(I,J)=0.0
        GOTO112
      7 IF(STOR(I+1,J).LT.AST)GOTO6
        SURP(I,J)=STOR(I+1,J)-AST
        DEF(I,J)=0.0
        STOR(I+1,J)=AST
    112 CONTINUE
    111 COVER=STOR(13,J)
        WRITE(6,498)N
    498 FORMAT(1X,I5)
C WRITE ESTIMATES OF SURPLUS AND DEFICIT
C       WRITE(6,501)((SC(I,J),I=1,12),J=1,N)
C       WRITE(6,501)((SURP(I,J),I=1,12),J=1,N)
C 501 FORMAT(1X,12F8.3)
        X(1)=0.0
        DO 113 I=1,N
    113 X(I+1)=X(I)+1.0
C DETERMINE CHANGES IN LAKE LEVEL FOR GENERATED VALUES
        DO 120 K=1,6
        D=3.0
        NF=1
        READ(5,516)TEDA(K)
    516 FORMAT(F10.0)
```

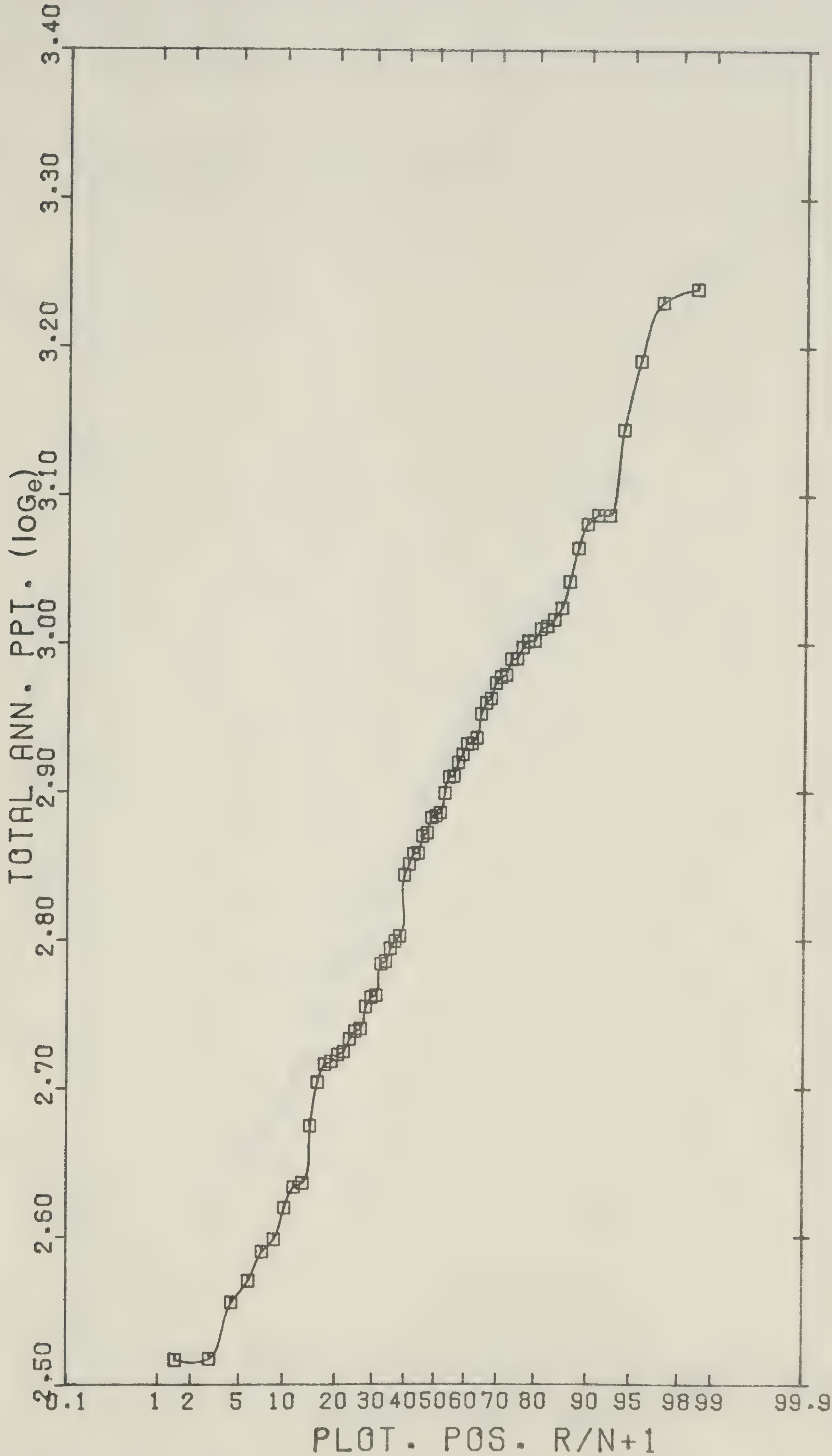
```
      READ(5,515)ALMAX(K),ALMIN(K),C(K),REST(K)
      READ(5,515)ALOW(K),AUP(K),BLOW(K),BHI(K)
  515 FORMAT(4F15.4)
      DO 114 J=1,N
      DO 119 I=1,12
      AL(I,J)=C(K)+SURP(I,J)*D/100.0  +SC(I,J)/100.0
      IF(AL(I,J).GE.ALMAX(K))GOTO230
      IF(AL(I,J).LE.ALMIN(K))GOTO204
      CHLI(I,J)=AL(I,J)-C(K)
      GOTO115
  230 CHLI(I,J)=ALMAX(K)-C(K)
      AL(I,J)=ALMAX(K)
      GOTO115
  204 CHLI(I,J)=0.0
      AL(I,J)=ALMIN(K)
  115 C(K)=AL(I,J)
      IF(C(K).GE.REST(K))GOTO232
      SA=ALOW(K)*C(K)-BLOW(K)
      GOTO323
  232 SA=AUP(K)*C(K)-BHI(K)
  323 D=(TEDA(K)-SA)/SA
  119 CONTINUE
  114 CONTINUE
      DO 116 J=1,N
      SCHIL(J)=0.0
      DO 117 I=1,12
  117 SCHIL(J)=SCHIL(J)+CHIL(I,J)
  116 CONTINUE
      DO 118 J=1,N
  118 B(J)=AL(MON,J)
C PLOT LAKE LEVELS FOR A PARTICULAR MONTH OF THE YEAR
      CALL CGPL(X,B,X,N,NF,1,1,4,1,0.0,50.0,10.0,
     *      ALMIN(K),2.0,5.0,ALPHA,6)
  120 CONTINUE
      CALL CGPL(X,B,X,N,0,1,1,4,1,0.,0.,0.,0.,0.,0.,ALPHA,6)
      STOP
      END
      SUBROUTINE GAUSS(IX,S,AM,V)
      A=0.0
      DO 50 I=1,12
      CALL RANDU(IX,IY,Y)
      IX=IY
   50 A=A+Y
      V=(A-6.0)*S+AM
      RETURN
      END
      SUBROUTINE RANDU(IX,IY,YFL)
      IY=IX*65539
      IF(IY)5,6,6
    5 IY=IY+2147483647+1
    6 YFL=FLOAT(IY)
      YFL=YFL*.4656613E-9
      RETURN
      END
END OF FILE
```

APPENDIX 3

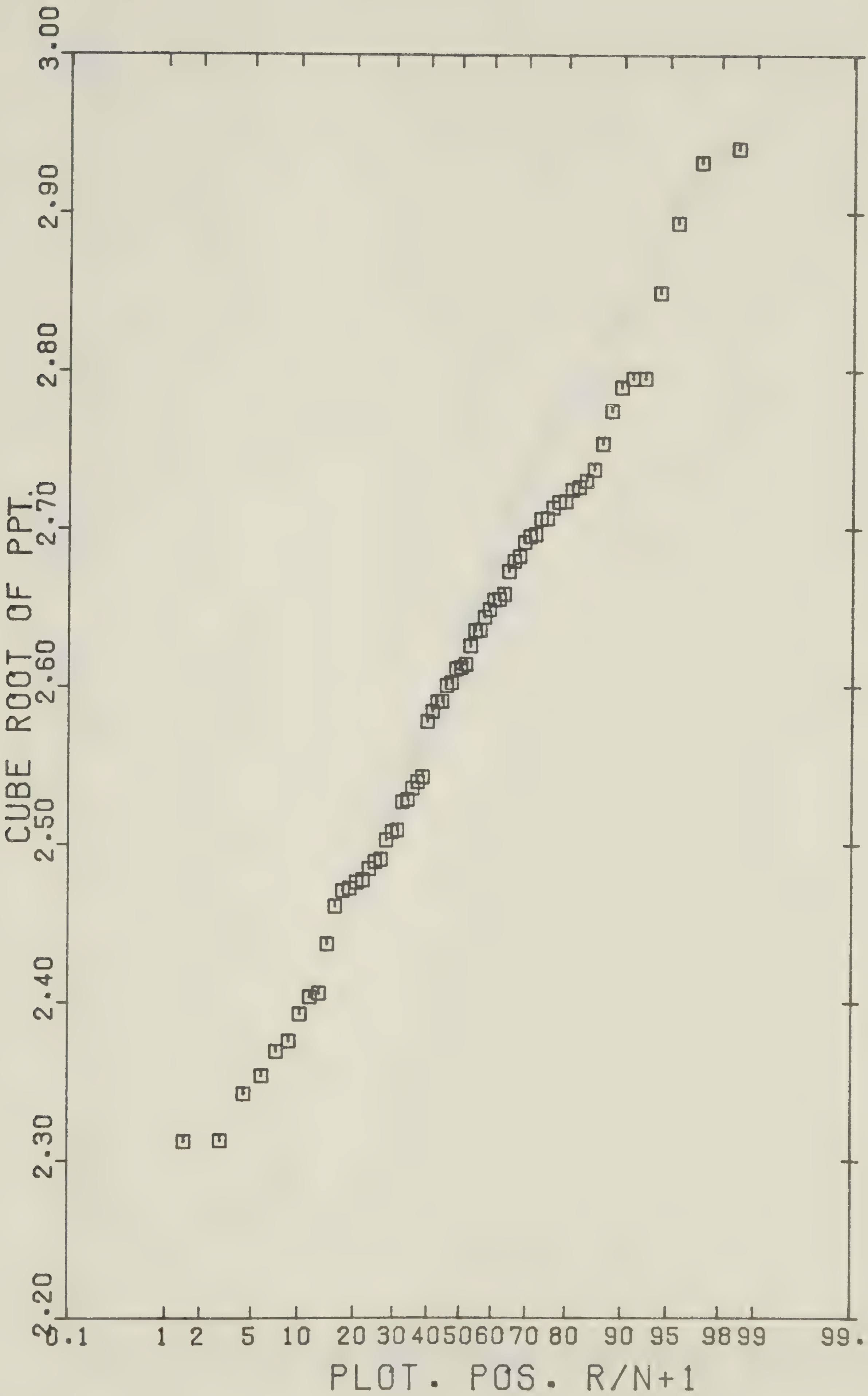
EXAMPLE FREQUENCY PLOTS



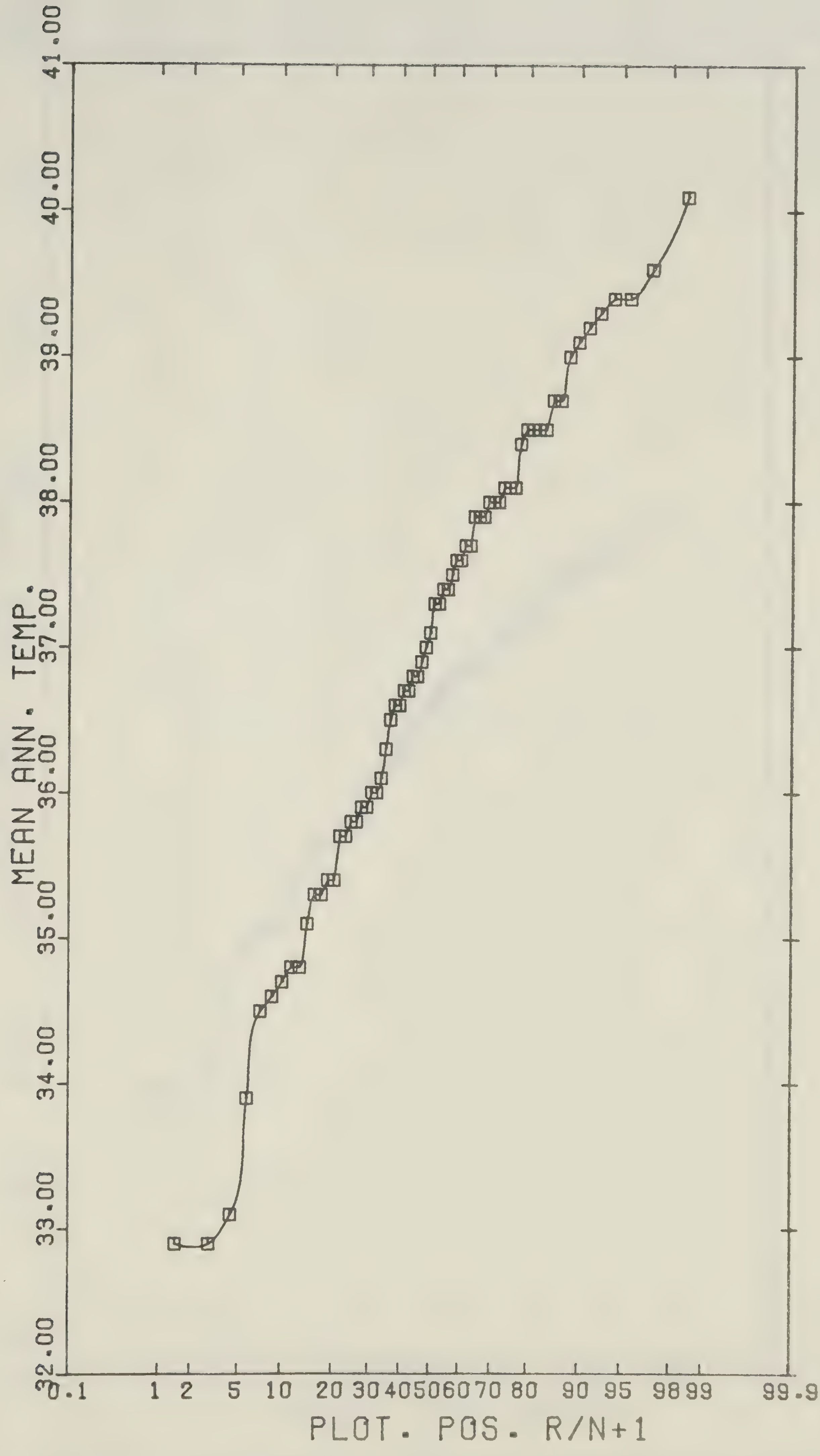
TOTAL ANNUAL PPT. VS. PLOT. POSITION R/N+1



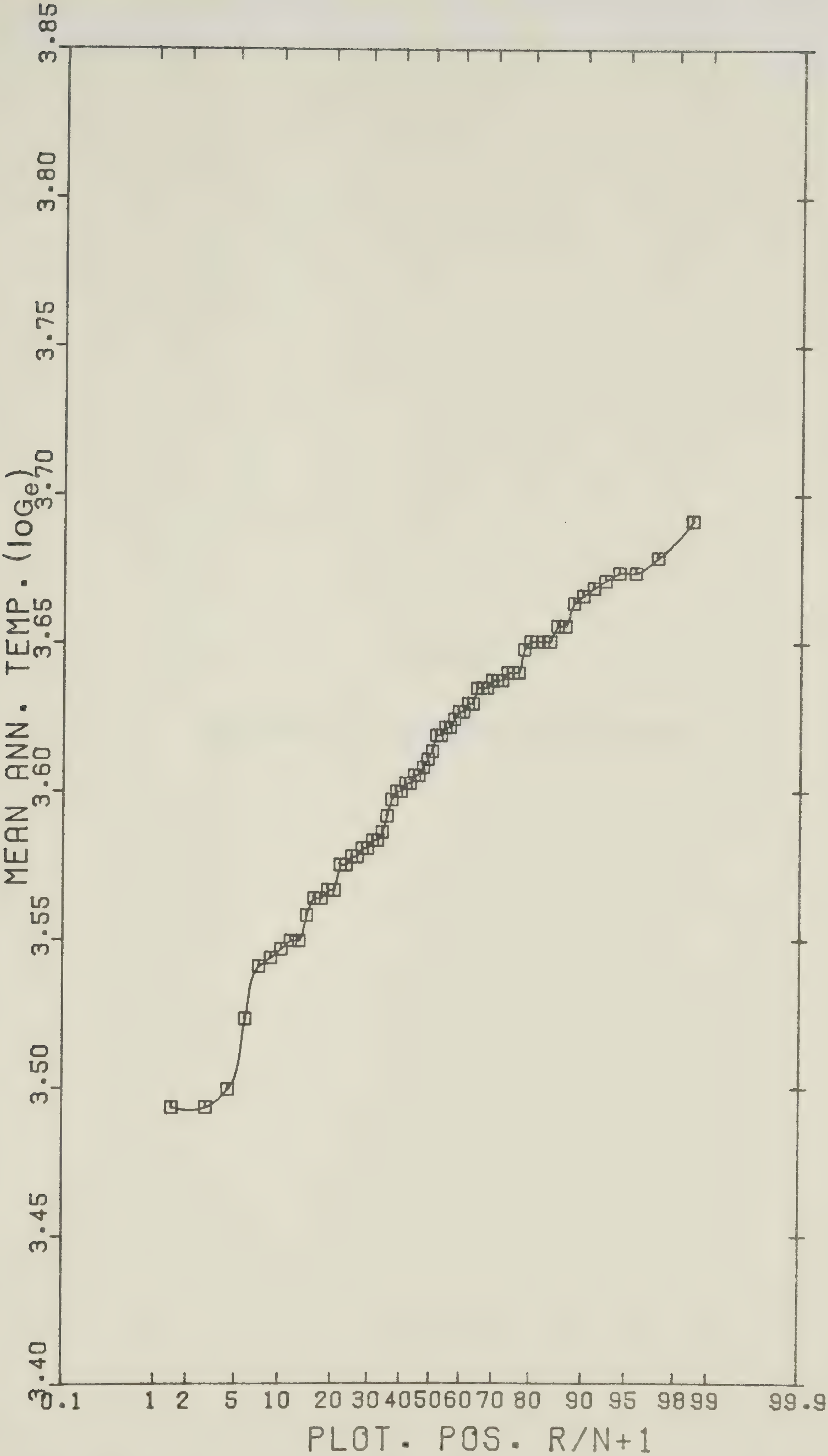
TOTAL ANNUAL PPT. VS. PLOT. POSITION R/N+1



CUBE ROOT OF ANN. PPT VS PLOTTING POSITION



MEAN ANNUAL TEMP. VS. PLOT. POSITION R/N+1



MEAN ANNUAL TEMP. VS. PLOT. POSITION R/N+1

APPENDIX 4

LAKE LEVELS OF OTHER LAKES IN THE SYSTEM

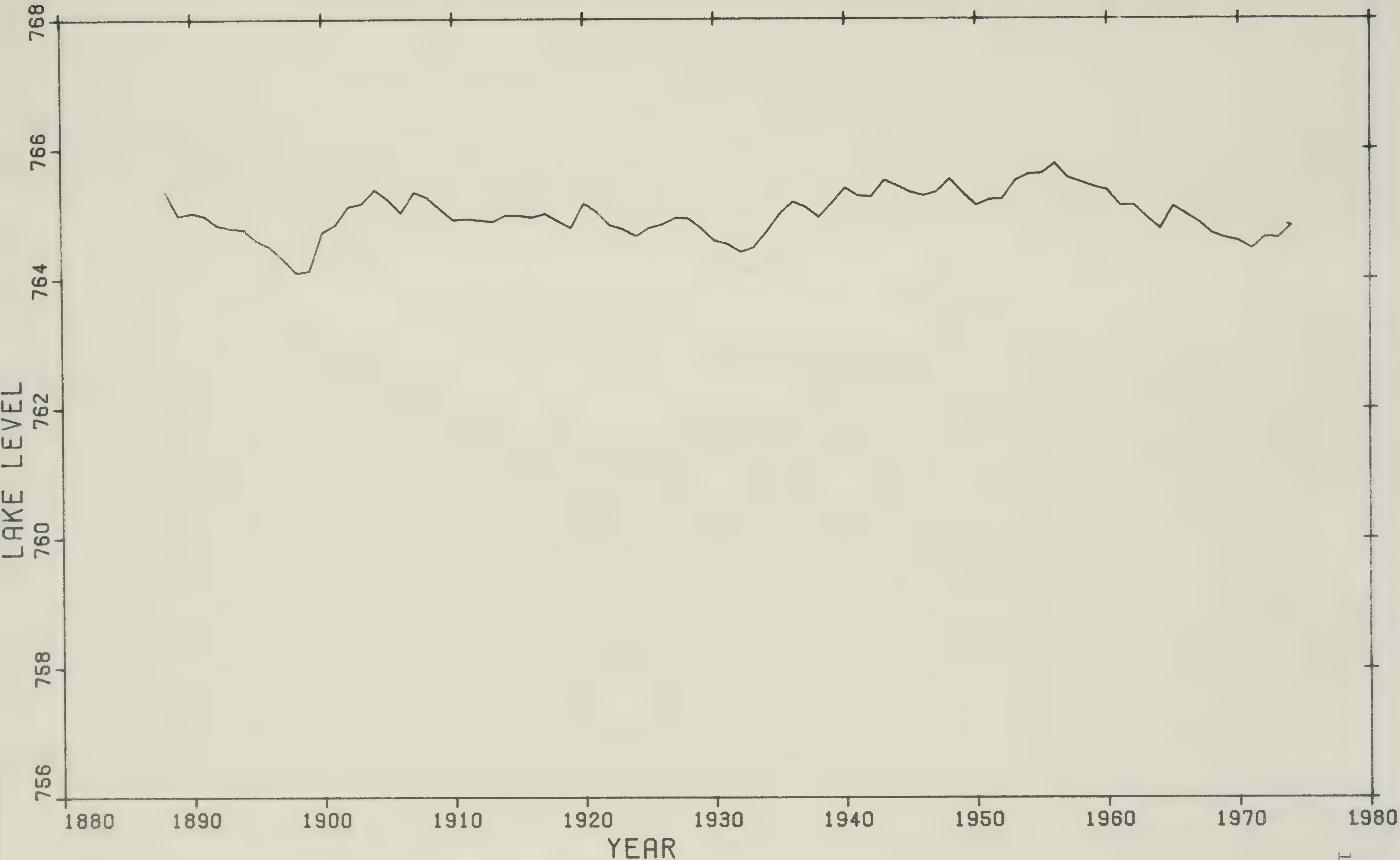


FIGURE (a) Miquelon Lake (F = 0.65)

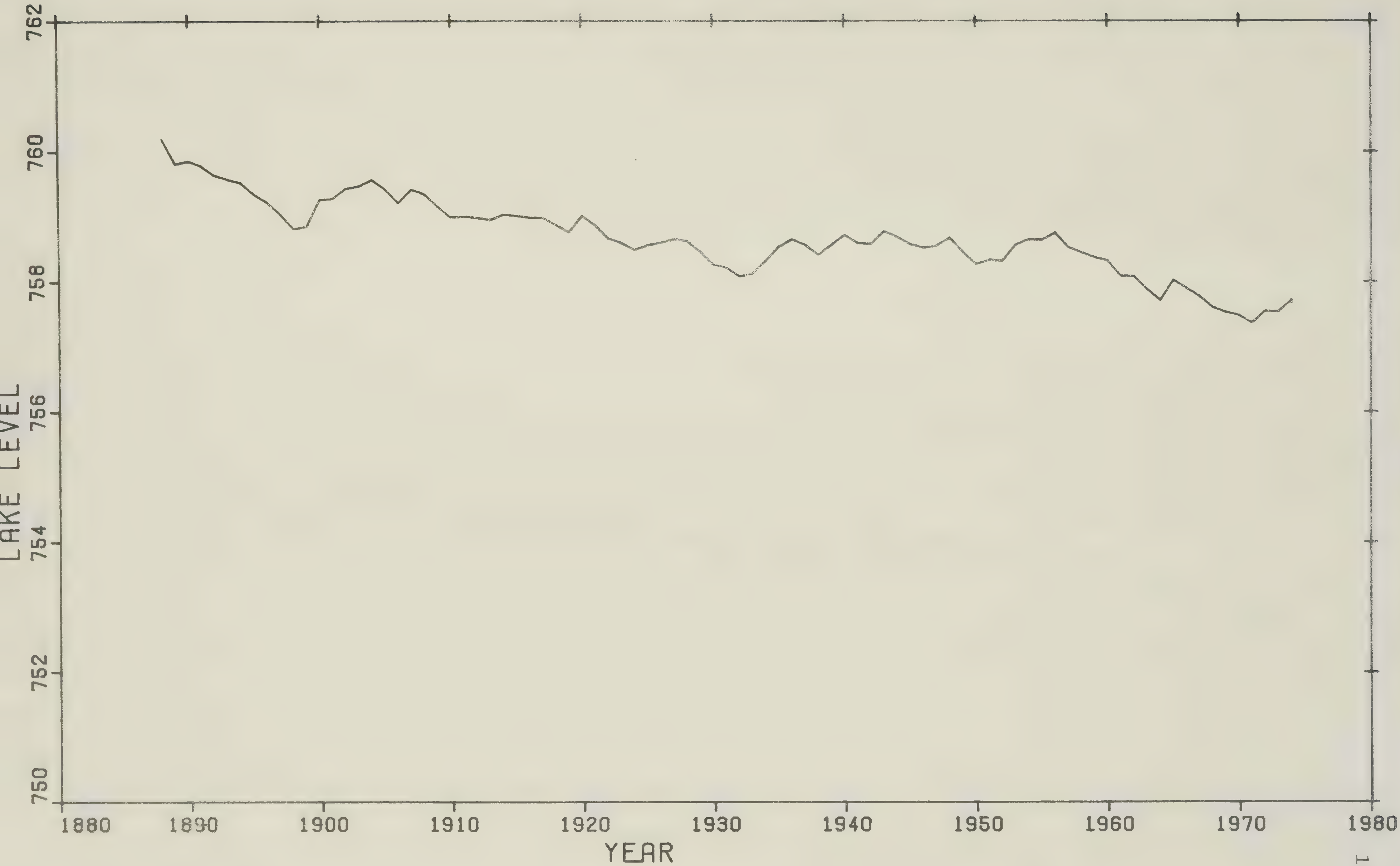


FIGURE (d) Ministik Lake (F = 0.65)

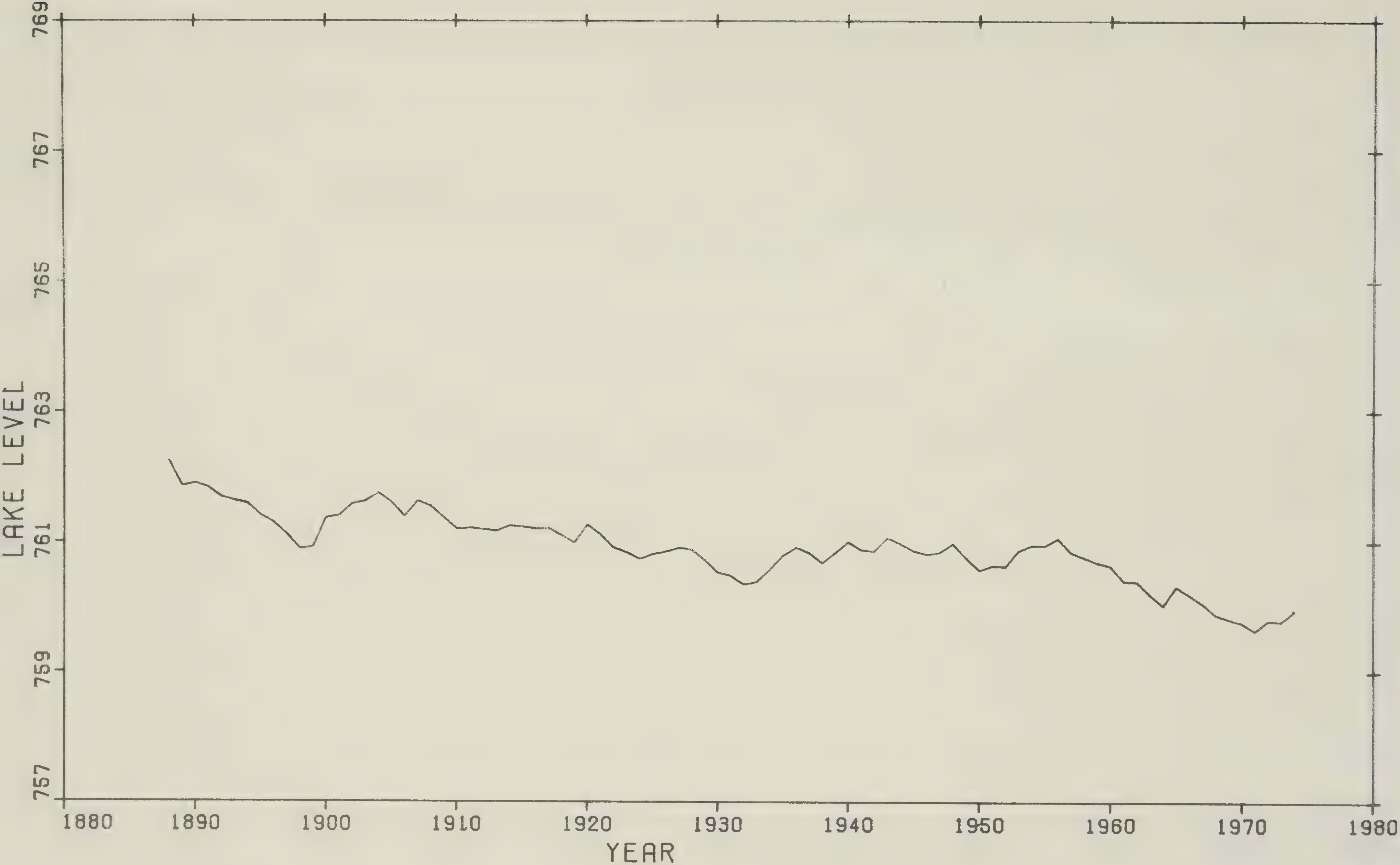


FIGURE (c) Oliver Lake (F = 0.65)

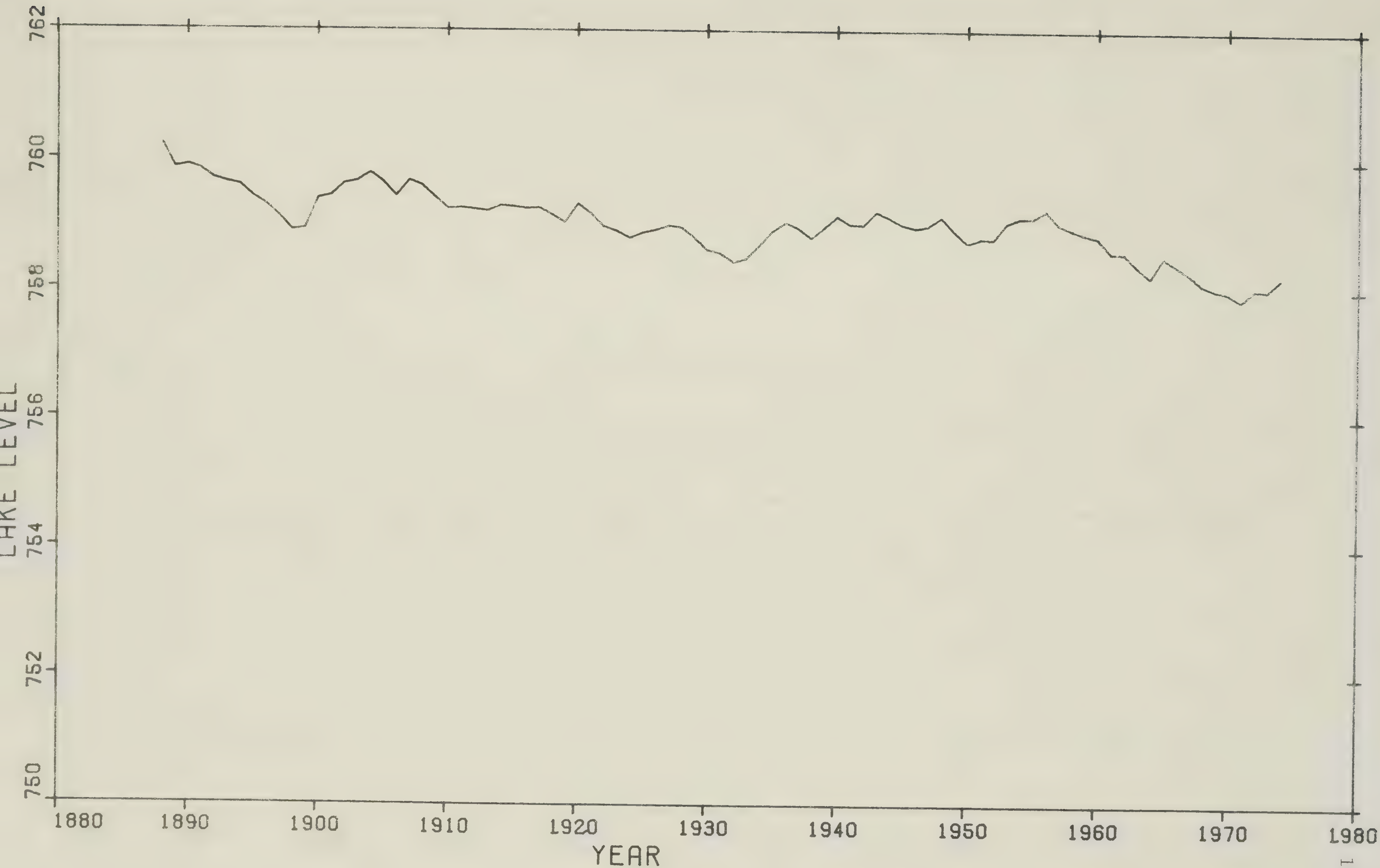


FIGURE (b) Joseph Lake (F = 0.65)

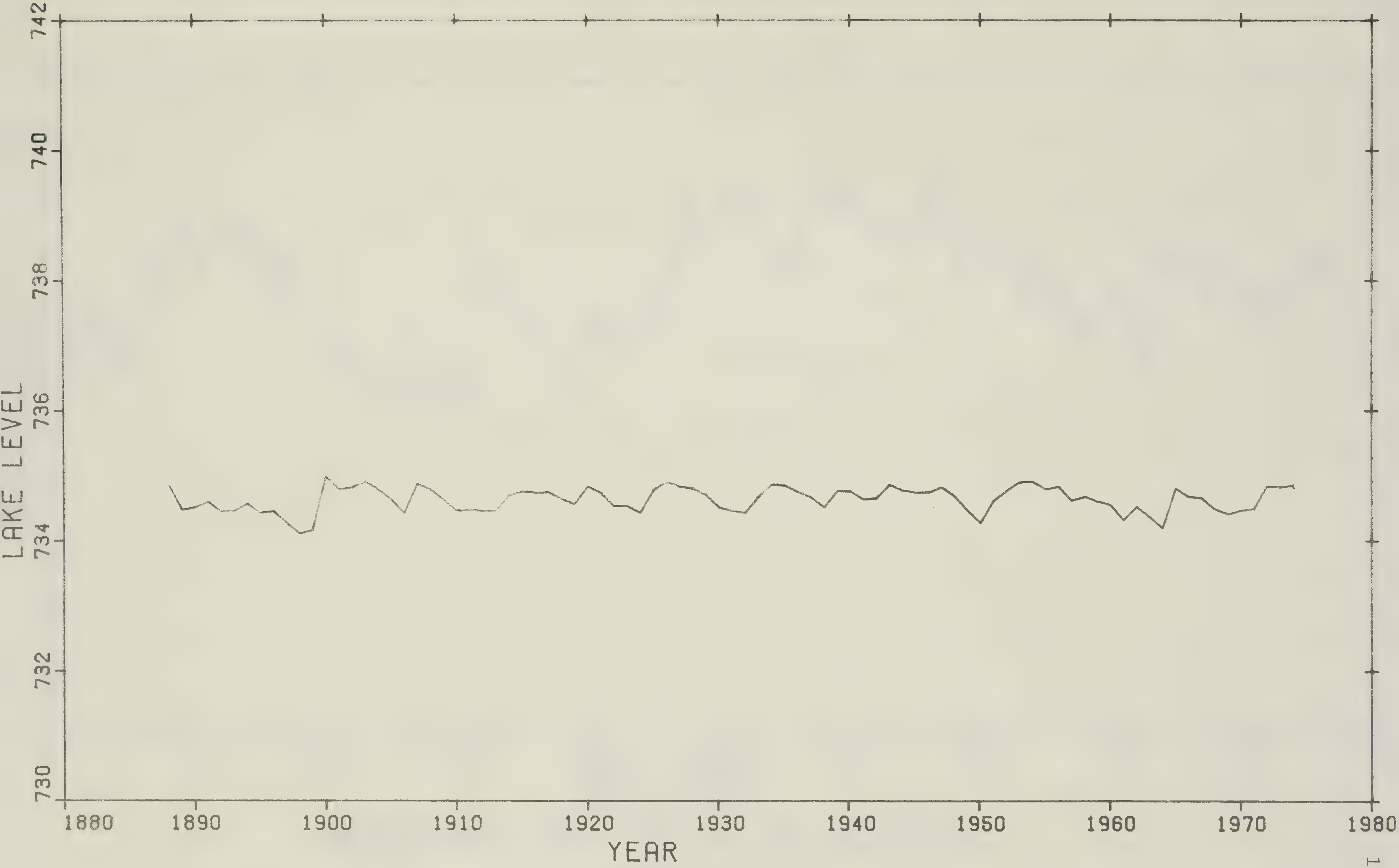
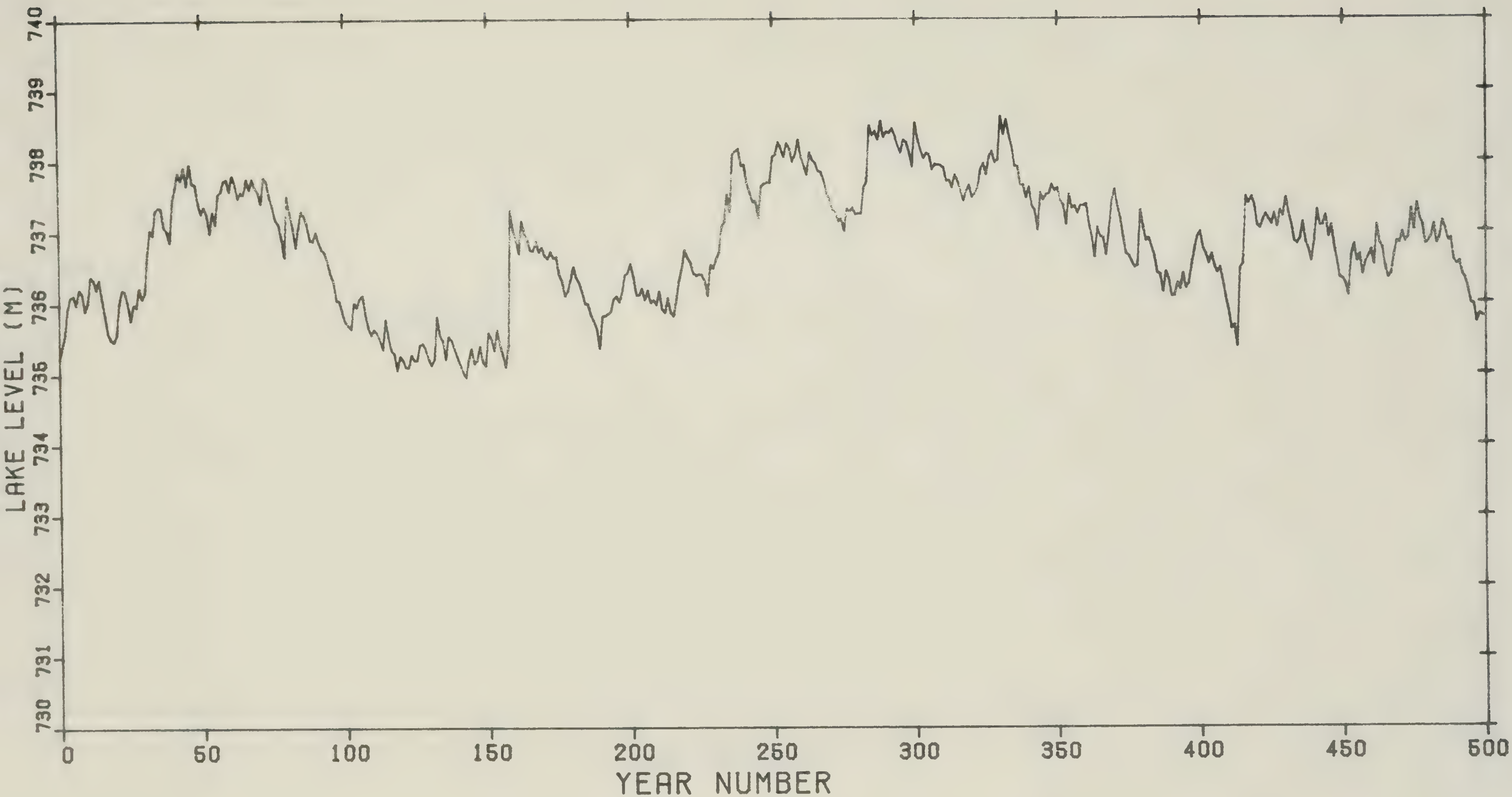
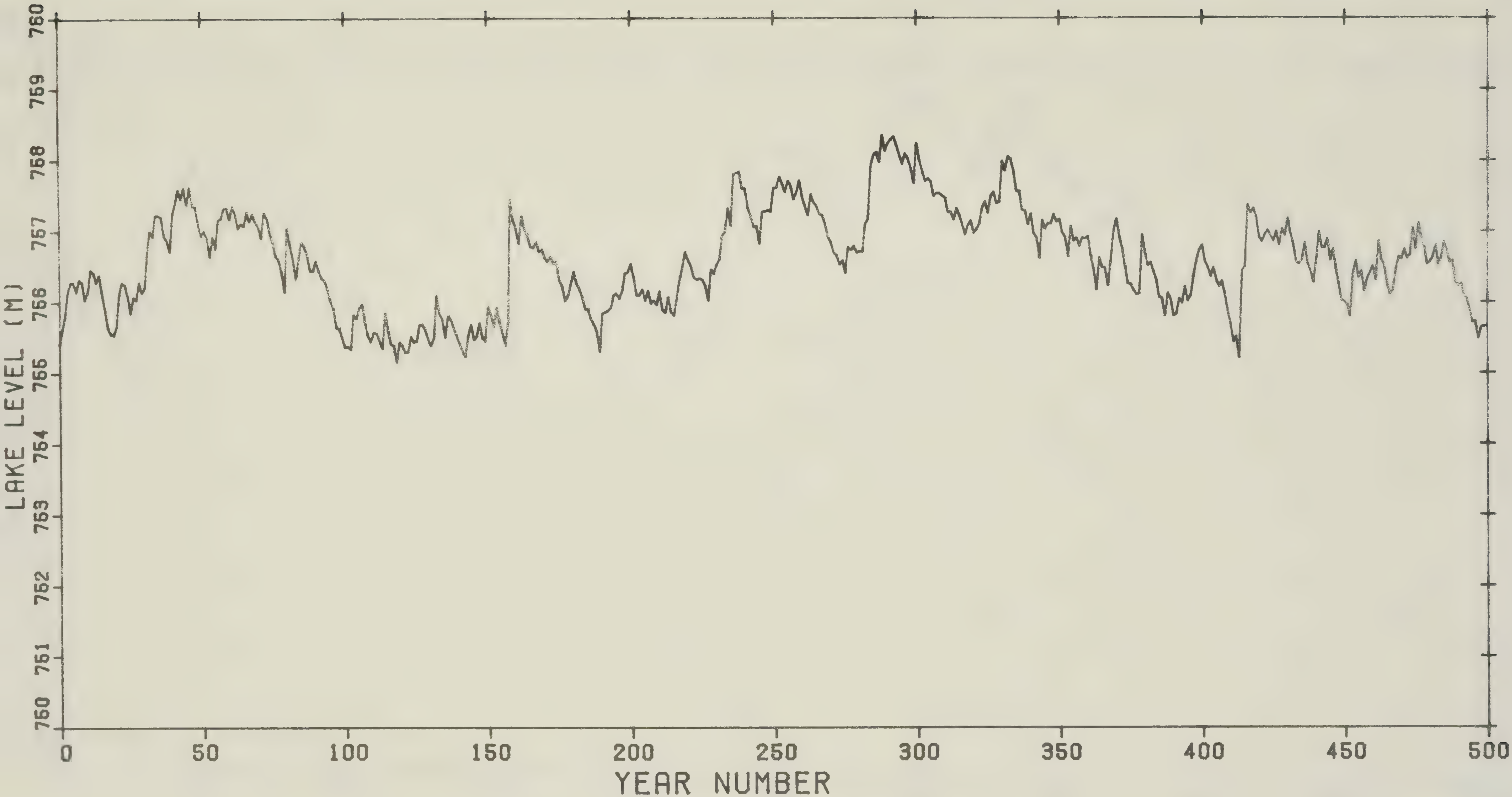


FIGURE (e) Hastings Lake (F = 0.65)



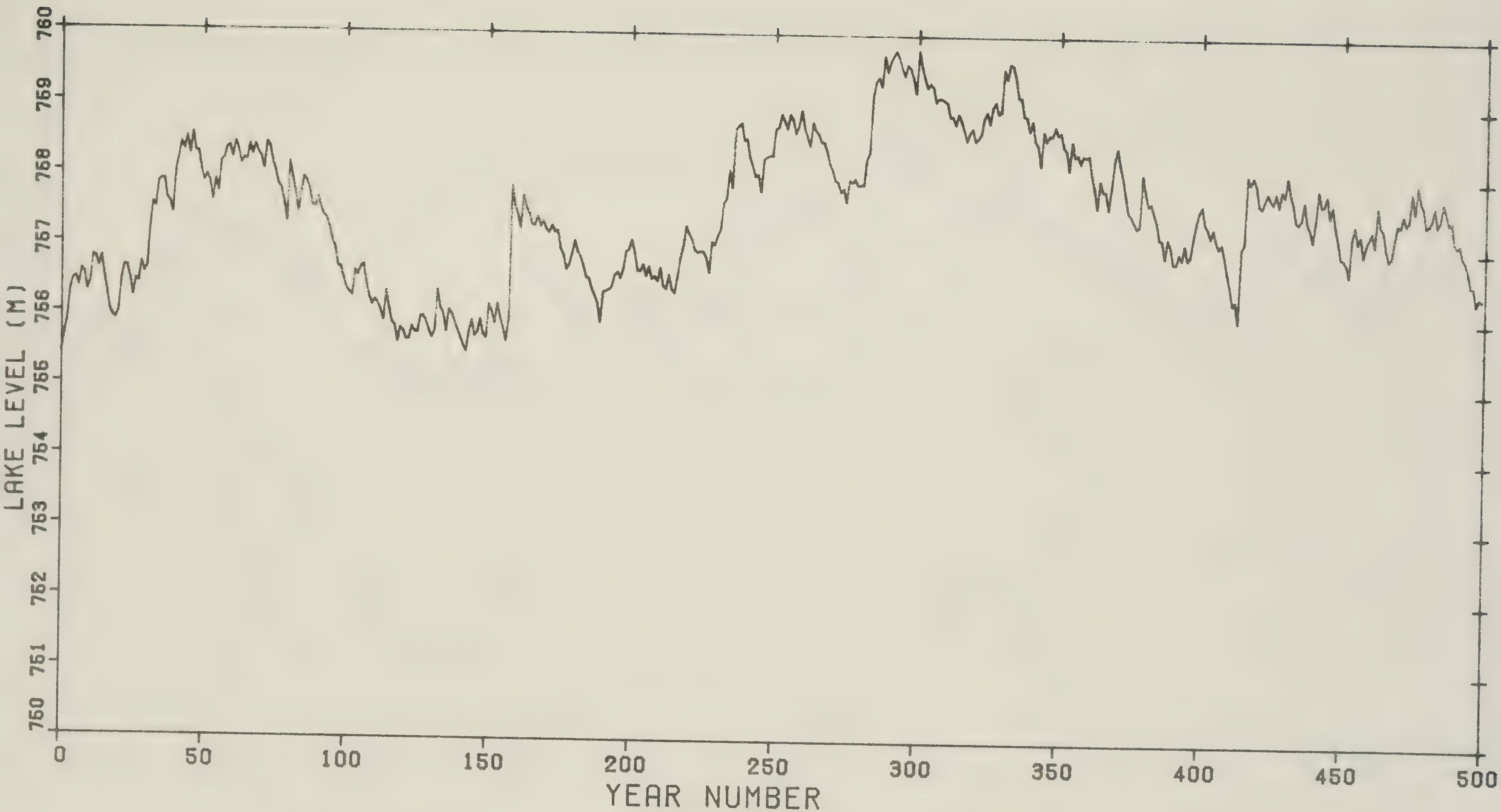
GENERATED LAKE LEVELS FOR OCTOBER

FIGURE (j) HASTINGS



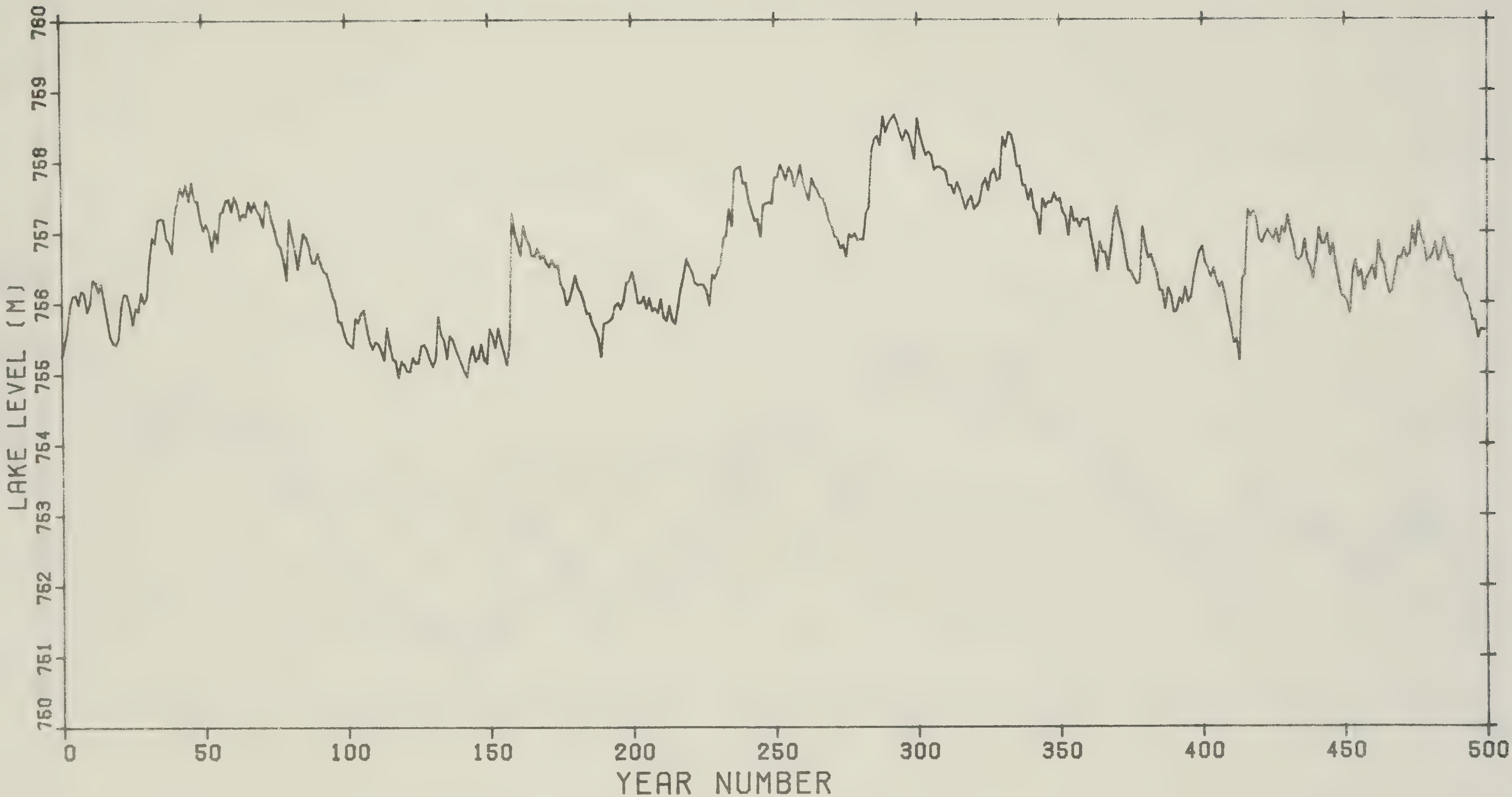
GENERATED LAKE LEVELS FOR OCTOBER

FIGURE (i) MINISTIK



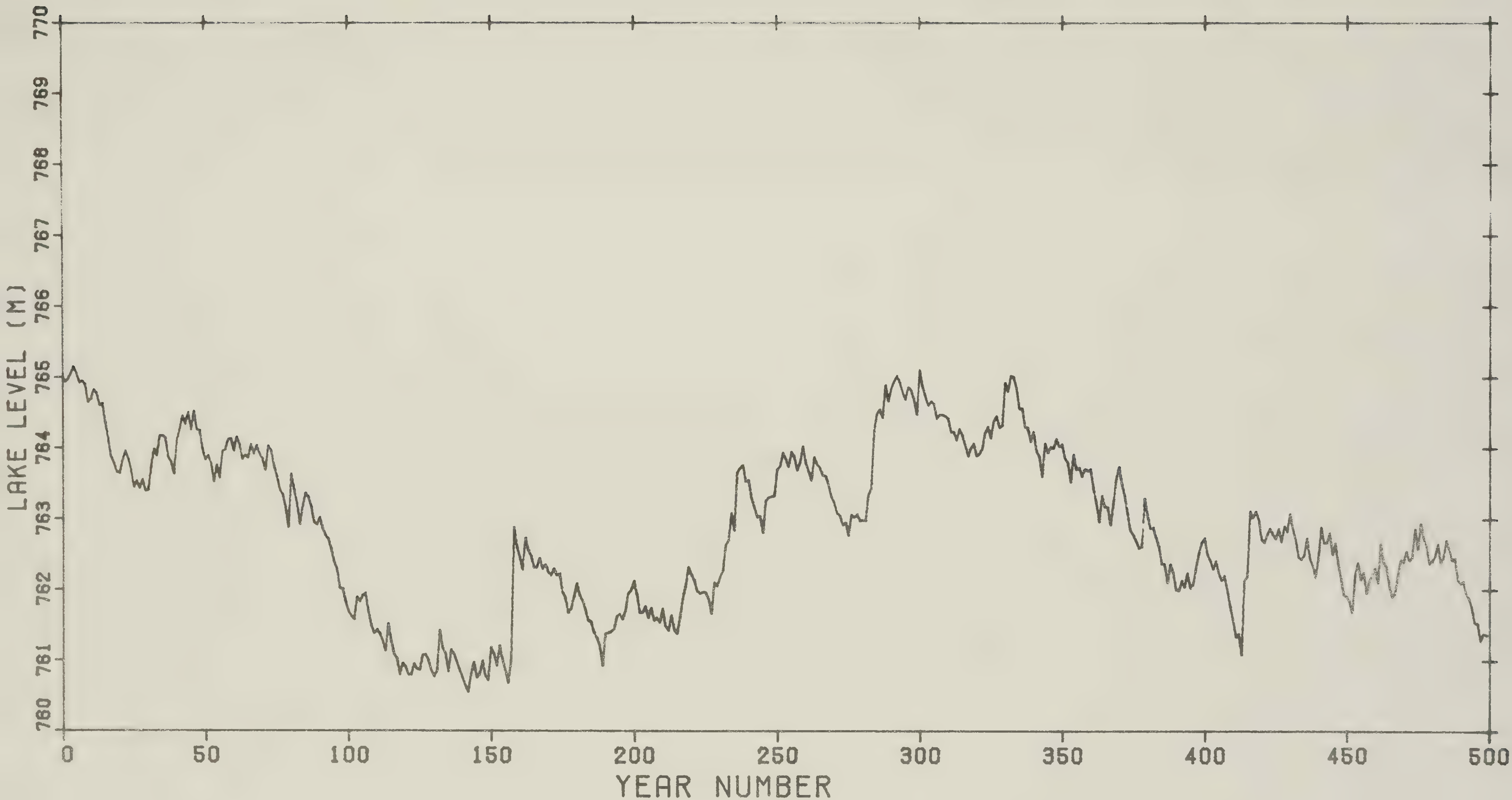
GENERATED LAKE LEVELS FOR OCTOBER

FIGURE (h) OLIVER



GENERATED LAKE LEVELS FOR OCTOBER

FIGURE (g) JOSEPH



GENERATED LAKE LEVELS FOR OCTOBER

FIGURE (f) MIQUELON